KINEMATICS OF MECHANISM WITH ROTATING CAM AND FLAT FACE OSCILLATING FOLLOWER. PART I: ANALYTICAL SOLUTIONS

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Abstract: The paper aims to analyze the kinematics of the mechanism with rotating cam and oscillating flat face follower. Compared to the knife edge follower, which has a fixed contact point on the follower, the flat face follower presents a mobile contact point with respect to the follower, with complex kinematics. Additionally, the tangency constraint between cam and follower makes difficult the position analysis. The analysis is made in two manners: the geometrical restraint method and the vector-contours method. Both methods offer the same solution, but the actual expressions differ considerably. There are obtained the position angle, the angular velocity and angular acceleration of the follower and the position, velocity and acceleration of the contact point with respect to the follower.

Keywords: cam mechanism, flat face oscillating follower, kinematic analysis

1. INTRODUCTION

The cam mechanisms are widely used in technical applications due to constructive minimalism and to the fact that the follower can execute any law of motion by an adequate design of the cam profile. There are numerous reference works in literature, presenting the analysis of cam mechanisms according to different criteria [1-7]. However, the number of papers dedicated to the study of cam mechanisms increases continuously due to the new methods employed in the analysis of mechanical systems, to the occurrence of new machining technologies and to the new restrictions imposed to mechanical systems.

There is a large diversity of kinematics analysis methods, starting with graphical and graphical-analytic techniques that are extremely expedite and applicable mainly to plane mechanisms and ending with analytical methods, appropriate to any spatial mechanism. Thus, Nikravesh’s approach, [8], concerning cam mechanisms is made as for any other mechanism required to obey certain geometrical constraints related to higher pair presence. Angeles applies the screw theory for the study of spatial mechanisms with cams which is among most modern methods used in robot kinematics.

The kinematics analysis of a cam mechanism in the case when the cam has a random shape, leads to intricate equations that can be solved by applying numerical calculus. With the aim of obtaining analytical closed forms, that could be the subject of discussions and interpretations, the subsequent analysis considers a circular eccentric as cam. Even for this very simple case, applying one or the other of the analysis methods, though conduct to the same result, the forms these results are obtained can be more or less complicated. In addition, one insists on the conditions for the existence of functions occurring in the expressions of solutions, these conditions being in fact the conditions ensuring the possibility of construction and working of the mechanism.

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2. GEOMETRIC LOCI INTERSECTION METHOD

Figure 1 presents the mechanism with rotating cam and oscillating flat face follower. The constructive parameters of the mechanisms are: $r$ - the radius of the disc from which the cam is made; $d$ - the assembly eccentricity of the disc; $L$ - the distance between the cam and follower joints.

In order to find the angle $\phi$ corresponding to the case the follower is tangent to the cam, the intersection points between the circle $\Gamma$ and the straight line $\Delta$ (Figure 1) are found and the condition to be identical is imposed.

\[
\begin{align*}
(x - d \cos \varphi)^2 + (y - d \sin \varphi)^2 &= r^2 \\
y &= -(x - L) \cdot \tan \psi
\end{align*}
\]

(1)

Fig. 1. Rotating cam mechanism with oscillating flat face follower.

The system (1) was solved using the MATHCAD utilitarian and the solutions obtained for the unknown $x$ have the expressions:

\[
x_{1,2} = \left[ \frac{d \cos \varphi + L \tan^2 \psi - d \sin \varphi \tan \psi \pm \sqrt{2Ld \cos \varphi \tan^2 \psi - 2d^2 \sin \varphi \cos \varphi \tan \psi - L^2 \tan^2 \psi + L + \sqrt{2} \left[ \frac{d \sin (\varphi + \psi) - L \sin \psi}{\cos \psi} \right]^2}}{2Ld \cos \varphi \tan \psi - d^2 \sin^2 \psi - d^2 \cos^2 \varphi \tan^2 \psi + r^2 - r^2 \tan^2 \psi} \right] \cdot \cos^2 \psi
\]

(2)

The MATHCAD program could not provide a simpler form of expressions from equation (2), therefore, after complicated and laborious calculus the expressions were brought to the final form:

\[
x_{1,2} (\varphi, \psi) = \left[ \frac{d \cos (\varphi + \psi) + \frac{\sin^2 \psi}{\cos \psi} L \pm \sqrt{r^2 - [d \sin (\varphi + \psi) - L \sin \psi]^2}}{\cos \psi} \right] \cdot \cos \psi
\]

(3)

For obtaining identical solutions, it is required that the quantity underneath the radical sign must be zero.

\[
r^2 - [d \sin (\varphi + \psi) - L \sin \psi]^2 = 0
\]

(4)

The equation (4) gives the angle $\psi$ for which the follower is tangent to the profile of the cam. These solutions are:
The relations (5) show the existence of two solutions for system (1), correspondent to two tangents that can be traced from an external point to a circle. In order to trace the position of the follower, it is necessary that the angles (4) to be real, namely, to impose positive sign for the radicand:

\[ L^2 + d^2 - 2dL \cos \phi - r^2 \geq 0 \]  

otherwise:

\[ \sqrt{L^2 + d^2 - 2dL \cos \phi} \geq r \]  

The condition (7) is the mathematical expression for the constraint that the distance AC, from the follower joint to the centre of the cam should be greater than the cam’s radius or the condition that the joint of the follower should be exterior the circular profile of the cam, since the tangents to a circle can be drawn only from external points with respect to the circle. By substitution of expressions (5) into relations (3), the abscissae of the two tangency points are obtained and following that, using this values and the second equation (1) the ordinate are obtained. After a series of calculus, the expressions for contact points coordinates are:

\[
\begin{cases}
  x_{1,2}(\phi) = d \cos[\psi_{1,2}(\phi) + \phi] \cos \psi_{1,2}(\phi) + L \sin^2 \psi_{1,2}(\phi) \\
  y_{1,2}(\phi) = -d \cos[\psi_{1,2}(\phi) + \phi] \sin \psi_{1,2}(\phi) + L \cos \psi_{1,2}(\phi) \sin \psi_{1,2}(\phi)
\end{cases}
\]  

From the two solutions obtained, the one corresponding to the assembly position for actual mechanism must be chosen.

Having the coordinates of the cam-follower contact point, the distance \( \lambda = CB \) from the follower’s joint to the contact point can be found:

\[ \lambda(\phi) = \sqrt{[x_{1,2}(\phi) - L]^2 + y_{1,2}^2} \]  

By substituting expressions (8) in (9) and after simplification, the following relation is obtained:

\[ \lambda(\phi) = \lambda_{1,2}(\phi) = L \cos[\psi_{1,2}(\phi)] - d \cos[\psi_{1,2}(\phi) + \phi] \]  

The two lengths are equal and so the index was omitted. The relation (10) is helpful because allows finding the minimum length of the follower as maximum value of it. Another usefulness of relation (10) consists in the fact that its derivative permits finding the relative velocity from the higher pair required in lubricant selection.

In Figure 2, there are presented the position angle of the follower \( \psi_{1,2} \), (Figure 2.a) the angular velocity of the follower (Figure 2.b), and the angular acceleration of the follower (Figure 2.c) together with the distance \( \lambda \) between the cam’s joint and the higher pair (Figure 2.d), the velocity (Figure 2.e) and acceleration (Figure 2.f), respectively, of the contact point alongside the follower.
a) Position angle of the follower.  

b) Angular velocity of the follower.  
c) Angular acceleration of the follower.  
d) Distance between the cam’s joint and the higher pair.  
e) Velocity of the contact point along the follower.  
f) Acceleration of the contact point along the follower.

3. VECTOR CONTOUR METHOD

The vector contour method is an analytical method that permits complete kinematical analysis of whichever mechanisms with lower pairs. With the intention of applying this methodology for the case of higher pairs mechanisms, it is required that, previously, all higher pairs should be replaced [9]. Generally, the replacement has an instantaneous character due to the variations of cam’s radius of curvature. In the present case, the substitution has a permanent nature due to the circular profile of the cam.

Figure 3.a presents the equivalent mechanism (dotted line) and the actual mechanism. The cam-follower pair was replaced by a fictive element and two pairs, a revolute pair, placed in the centre of the cam, and a prismatic pair.
in the contact point, with the possibility of translational motion along the follower’s direction. Obviously, the length of the replacing element equals the radius of the cam.

Figure 3.b presents the vector contour associated to the equivalent mechanism. The equation of the closed vector contour is:

\[ I_1 + I_2 + I_3 + I_0 = 0 \]  

(11)

The equations projected on coordinate axes are:

\[
\begin{cases}
I_1 \cos \varphi_1 + I_2 \cos \varphi_2 + I_3 \cos \varphi_3 + I_0 \cos \varphi_0 = 0 \\
I_1 \sin \varphi_1 + I_2 \sin \varphi_2 + I_3 \sin \varphi_3 + I_0 \sin \varphi_0 = 0.
\end{cases}
\]

(12)

From Figure 3.b, it results:

\[ I_1 = d, \ \varphi_1 = \varphi, \ I_2 = r, \ \varphi_2 = \pi/2 - \psi, \ I_3 = \lambda, \ \varphi_3 = 2\pi - \psi, \ I_0 = L, \ \varphi_0 = \pi \]

(13)

Substituting relations (13) into (12), it results:

\[
\begin{cases}
d \cos \varphi + r \sin \psi + \lambda \cos \psi - L = 0 \\
d \sin \psi + r \cos \psi - \lambda \sin \psi = 0
\end{cases}
\]

(14)

The unknown of the system are the position angle of the follower \( \psi \) and the length \( \lambda \) that defines the position of the contact point alongside the follower. By separating in both equations the terms containing the unknown \( \lambda \) and effectuating the ratio between equations member by member, it is obtained an equation that permits finding the angle \( \psi \) as function of angle \( \varphi \):

\[
\left( d \sin \varphi + r \cos \psi \right) / \left( d \cos \varphi + r \sin \psi - L \right) = -\tan \psi
\]

(15)

The values for the solutions of equations (13) and (15) for the same mechanism are presented comparatively:

\[
\begin{align*}
\text{atan} & \left( \frac{d \sin \varphi - d^2 \sin \varphi \cos \varphi \pm \sqrt{L^2 + d^2 - r^2 - 2dL \cos \varphi}}{(L - d \cos \varphi)^2 - r^2} \right) = \left[ \begin{array}{c} 0.767 \\ -0.387 \end{array} \right] \\
2 \cdot \text{atan} & \left( \frac{L - d \cos \varphi \pm \sqrt{L^2 + d^2 - r^2 - 2dL \cos \varphi}}{r - d \sin \varphi} \right) = \left[ \begin{array}{c} 0.767 \\ -0.387 \end{array} \right]
\end{align*}
\]

(16)

It results an equation of a very simple form, with \( \lambda \) unknown:

\[ \lambda^2 + r^2 - L^2 - d^2 + 2dL \cos \varphi = 0, \]

(17)

having the solutions:

\[ \lambda_{1,2} = \pm \sqrt{L^2 + d^2 - 2dL \cos \varphi - r^2}. \]

(18)

From the two solutions, considering the physical significance, only the positive one must be considered. It can be immediately be verified that the relations (10) and (18) are the same, but one must remark the simplicity of the last one. The uncomplicated form of relation (18) allows finding the extreme values of the unknown \( \lambda \):
\[
\lambda_{\text{max}} = \sqrt{L^2 + d^2 + 2dl - r^2} = \sqrt{(L + d)^2 - r^2}, \quad \lambda_{\text{min}} = \sqrt{L^2 + d^2 - 2dl - r^2} = \sqrt{(L - d)^2 - r^2}
\] (19)

In Figure 4 there were plotted the trajectories of the contact point for the two mechanisms that can be constructed, in the reference system of the ground, and the hodograph of relative velocity:

![Fig. 4. Trajectory of cam-follower contact point and the hodograph of relative velocity from the higher pair.](image)

4. CONCLUSIONS

The paper analyses from kinematical point of view the mechanism with rotating cam and oscillatory flat face follower. The tangency condition between the profile of the cam and the profile of the follower is a supplementary one, compared to the mechanism with knife edge follower. In the case of knife edge follower, the absolute trajectory is an arc of circle with the centre in the joint of the follower while for the flat face follower the absolute trajectory is an unsystematic curve, a function of contact point displacement alongside the follower. The kinematics parameters, position, velocity, acceleration, characteristic to absolute motion of the follower and for the relative motion of contact point were obtained by two different study methods. Obviously, the methods conduct to the same results but due to presence, in the expressions obtained, of trigonometry functions, the actual forms of the solutions differ and it is quite difficult to pass from an expression to another. This affirmation is validated by the fact that the expressions for position parameters were obtained using the same software. Requiring the software to simplify the two expressions, two different expressions were offered and one cannot state that the expressions are equivalent.

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