CALCULATION OF LOCAL STRESS STATE FOR SHELLS INTERSECTION

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Abstract: This paper presents current calculation methods in order to determine the state of stress in the nozzle – shell junction section, their peculiarities, and limitations. These can be applied only to the linear-elastic behavior of the materials components. In the case of nonlinear behavior, beyond the yield stress limit, there is proposed a calculation method based on the concept of specific energy participation.

Keywords: shells intersection, state of stress, junction calculation method, critical energy principle.

1. INTRODUCTION

In general, the pressure vessels are equipped with nozzles that usually connect removable pipes, various types of fittings and other components and their devices [1]. From the point of view of the position to the reference surface (median, outer etc.) of the vessel type structure (geometric criterion), the nozzles can be:
- perpendicular or radial, when their geometrical axis is identified with the normal to the nominated surface, (Figure 1. a and b);
- inclined or oblique, when their geometrical axis is inclined at a certain angle to the normal to the nominated surface, being in the same longitudinal plane of symmetry with the latter (Figure 1. c and d);
- tangential when their geometric axis is in a transverse plane is inclined at a certain angle to the normal to the nominated surface or their geometrical axis is not in the same plane (longitudinal) of symmetry with the normal to the shell surface (Figure 1. e).

Fig. 1. Nozzle - shell intersections.
The state of stress in the nozzle-shell junction was investigated in the papers [2 - 6]. Consequently, some results were introduced into the calculation standards [7-10]. These norms, as well as some results in the literature, have drawbacks that will be shown below. The major drawback is that it only refers to the stresses below the yield stress. In fact, in the junction areas, stresses often exceed the yield stress.

The state of stress in a shell can be determined according to the theory without bending moments or the theory with bending moments. At stresses below the yield stress, the effects superposition can be done by the algebraic summation of the external loads effects [11, 12]. If the stress exceeds the yield stress, solving the superposition effects problems cannot be done by the algebraic summation of the individual effects of the different loads.

The paper analyzes the current state of the calculation methods for determining the state of stress at the nozzle-shell junction. Since all current methods refer only to the state of stress in the linear -elastic field, the paper goes on to propose a calculation method applicable to the non-linear field, beyond the yield stress.

To this end, we resort to the principle of critical energy, and we use the specific energy participation concept [13 - 15].

2. GENERAL ISSUES OF THE NOZZLES – SHELLS INTERSECTION

2.1. Bijlaard’s research results

The determination of the stress state in the nozzle-shell intersection was analyzed by several authors [2 - 6] and is included in some normative calculation [7-10]. These will be discussed below. In the work of P.P. Bijlaard [2] it is presented the theoretical basis for analyzing the stress state of the circular cylindrical shells using the Fourier double series. In the paper [3] Bijlaard determined the effect of the local loads acting on a spherical shell on which a radial load \( P \) is applied, intersected with a rigid cylindrical insert. Direct solutions for the spherical shells were obtained. The numerical results of the study are presented graphically. As a first approximation, Bijlaard assumes that the attachment can be idealized and replaced by a rigid cylinder (Figure 2).

![Fig. 2. Spherical shell rigid - cylinder junction.](image)

The following notations are introduced (Figure 2): \( r_0 \) is the outside radius of cylindrical nozzle; \( r \) is the radius of curvature of middle surface of shell section in a latitudinal plane; \( t \) is the thickness of the cylindrical nozzle; \( N_x \) is the normal membrane force in the shell wall in the radial directions; \( R \) is the mean radius of spherical shell; \( \varphi \) is the angle between normal to shell middle surface and shell axis.

Bijlaard neglected in the solutions of the differential equations the terms containing \( t^2/12r^2 \) as he considered the ratio \( t/r < 0.1 \), thus \( t^2/12r^2 < 0.001 \). Following this approximation, Bijlaard stated that the results could be with 25% lower than the calculated ones. Bijlaard [3] presented some numerical results for the radial load, with bending and torsion moment. In this way of treating there are two essential deficiencies: the omission
of the higher order terms in the obtained differential equation and load position restrictions. In addition, Bijlaard's solution can be applied only to points which are far from the ends of the shell.

2.2. Method of calculation according to The Welding Research Council 107 for the nozzle - spherical or cylindrical shell intersection

The method of The Welding Research Council 107 [4] is much more straightforward than that of Bijlaard. Based on a set of initial parameters of the external loads, one can determine the stresses and membrane forces. It also takes two stress concentration factors $K_n$ and $K_b$ that are based on the size of the intersected elements. Once these values are known, the stress is calculated by relations of the form:

$$\sigma = K_n \cdot \left( \frac{N}{T} \right) \pm K_b \cdot \left( \frac{6M}{T^2} \right)$$

where $N$ is the normal membrane force in the shell wall on the radial ($N_x$) or circumferential ($N_\theta$) directions; $M$ is the bending moment in the spherical shell wall; $T$ is the thickness of spherical shell wall; $K_n$ is the membrane stress concentration factor (tension or compression); $K_b$ is the bending stress concentration factor.

The procedure in [4] contains the same deficiencies as in Bijlaard’s paper: there cannot be obtained the displacements or the interaction effect from several joints. The curves in the diagrams were represented for a given set of parameters, any other data that is not found on these curves requires interpolation or extrapolation.

a) The following is the calculation of the stresses for the spherical shells – nozzles (Figure 3) applied with a radial force $P$, with the bending moment $M$ and torsional moment $M_r$ the tensions and the transversal or shear force, $V$ in the spherical shell having the following expressions [4]:

- radial stresses,

$$\sigma_x = K_n \cdot \left( \frac{N_x}{T} \right) \pm K_b \frac{6M_x}{T^2}$$

- tangential stresses,
- stresses resulting from the torsional moment $M_r$ (Fig. 3),

$$\tau_{yz} = \tau_{xy} = \frac{M_r}{2\pi r_0^2 T}$$

(4)

- stresses resulting from the shear load $V$,

$$\tau_{xy} = \frac{V}{\pi r_0 T} \sin \theta$$

(5)

In the previous relations, $\theta$ is the angle around the nozzle; $R_m$ is the mean radius of spherical shell; $r_0$ is the outside radius of cylindrical nozzle; $T$ is the thickness of spherical shell; $t$ is the thickness of the cylindrical nozzle. The method allows the evaluation of the spherical shell stresses but not those of the nozzle. In practice, the nozzle stresses are often higher than those of the spherical shell.

b) It is considered the cylindrical shell with applied radial force $P$, with the transversal forces $V_C$ and $V_L$, the circumferential bending moment $M_C$, the meridional bending moment $M_L$ and the torsional (twisting) moment $M_r$ (Figure 4).

![Figure 4. Cylindrical shell (1) joint with a nozzle (2).]

The following notations are introduced (Figure 4): $V_C$, $V_L$ - the concentrated shear load in the circumferential and longitudinal direction, respectively of the shell 2; $M_C$, $M_L$ - the external bending moment in the circumferential and longitudinal direction, respectively of the shell 2; $l$ - length of the cylindrical shell 1; $x, y$ – the linear coordinates in the longitudinal and circumferential directions; $\varphi$ - cylindrical coordinate in the circumferential direction of the shell 1; $\alpha = l/R_m$; $\gamma = R_m/T$; $\beta$ - nozzle parameter; $\sigma_\varphi$, $\sigma_x$ - normal
stress in the circumferential and longitudinal directions, respectively; \( \tau_{ij} \) - shear stress on the \( i \) perpendicular on the \( j \) direction.

The stresses in the shell wall 1 are calculated by the following relations [4]:

- radial stress:

\[
\sigma_x = K_n \cdot \frac{N_x}{T} \pm K_h \frac{6M_x}{T^2}
\]  

(6)

- tangential stress:

\[
\sigma_\phi = K_n \cdot \frac{N_\phi}{T} \pm K_h \frac{6M_\phi}{T^2}
\]  

(7)

The relations are applied separately for \( P \), \( M_C \) and \( M_L \) and there results \( \sigma_x(P), \sigma_x(M_C) \) and \( \sigma_x(M_L) \) and also \( \sigma_\phi(P), \sigma_\phi(M_C) \) and \( \sigma_\phi(M_L) \). Finally, for the linear - elastic behavior of the shell material, the total stresses are calculated by algebraic summation with the relations:

- normal stresses,

\[
\begin{align*}
\sigma_x &= \sigma_x(P) + \sigma_x(M_C) + \sigma_x(M_L) \\
\sigma_\phi &= \sigma_\phi(P) + \sigma_\phi(M_C) + \sigma_\phi(M_L)
\end{align*}
\]  

(8)

- the tangential stress resulting from the torsional moment,

\[
\tau_{x\phi} = \tau_{x\phi} = \frac{M_r}{2\pi \cdot r_0^2 \cdot T}
\]  

(9)

- the shear stress due to the tangential circumferential force,

\[
\tau_{x\phi} = \frac{V_c}{\pi \cdot r_0 \cdot T} \cos \theta
\]  

(10)

- the shear stress due to the meridional circumferential force,

\[
\tau_{x\phi} = \frac{V_L}{\pi \cdot r_0 \cdot T} \sin \theta
\]  

(11)

In order to do the calculations, the loads \( M \) and \( V \) (Figure 3) are decomposed in two perpendicular planes: the bending moment \( M \) is decomposed into the components \( M_C \) and \( M_L \) (Figure 4). In the same way, \( V \), as the tangential force, decomposes into the components \( V_C \) and \( V_L \) (Figure 4).

2.3. Method of calculation according to the Welding Research Council Bulletin 297 for the nozzle – cylindrical shell intersection

The Calculation Bulletin 297 [5] is a supplement of the WRC 107 [4] which extends the coverage of WRC 107. In [5] the calculation refers to a cylindrical nozzle attached to a cylindrical shell. The tensions of both shells can be determined for a \( D/T \) ratio higher than in the WRC 107. The notations used for the nozzle – shell junction:
\( \sigma_r, \sigma_\theta \) - the tension in the radial and circumferential direction, respectively in the shell wall, (Figure 5); \( \sigma_a \) - the stress on axial direction of the nozzle.

![Figure 5](image)

Fig. 5. Positive directions of the external loads of the nozzle (2) intersected with the cylindrical shell (1).

Force \( P \), the bending moment components \( M_i \) (noted \( M_C \) and \( M_L \)), the shear force components \( V_i \) (noted \( V_C \) and \( V_L \)), and the torque \( M_r \) cause the following stresses [5]:

- in the cylindrical shell (1 in Figure 5), tangential stresses,

\[
\begin{align*}
\tau &= 2M_r/\left(\pi \cdot d^2 \cdot T\right) \\
\tau &= 2V_i/\left(\pi \cdot d \cdot T\right)
\end{align*}
\]

- in the nozzle (2 in Figure 5) at the junction with the shell, the axial stresses are,

\[
\begin{align*}
\sigma_a(P) &= \frac{P}{t^2}\left[\frac{t}{\pi \cdot d} \pm \left(6m_r - 3n_r\right)\right] \\
\sigma_a(M_i) &= \frac{M_i}{t^2d}\left[\frac{4t}{\pi \cdot d} \pm \left(6m_r - 3n_r\right)\right]
\end{align*}
\]

The circumferential stress in nozzle is considered equal to the circumferential stress in the shell at the nozzle – shell junction. This assumption is justified by the fact that the maximum stress appear at the junction of the shells and that the two shells have the same radial deformation. In the cylindrical shell 1 the circumferential stresses are,

\[
\begin{align*}
\sigma_\theta &= \frac{P}{T^2}(n_\theta) \\
\sigma_\theta &= \frac{M_i}{T^2d}(n_\theta)
\end{align*}
\]

In nozzle 2, the circumferential stresses at the junction with the shell 1 are,
\[
\begin{align*}
\tau &= 2M_r/\left(\pi \cdot d^2 \cdot t\right) \\
\tau &= 2V_i/\left(\pi \cdot d \cdot t\right)
\end{align*}
\]  
(15)

In the previous relations, the following notations were used:

- \( m_r = M_r/P \) or \( M_r \cdot d/M_c \) or \( M_r \cdot d/M_L \)
- \( n_r = N_r \cdot T/P \) or \( N_r \cdot Td/M_c \) or \( N_r \cdot Td/M_L \)
- \( m_\theta = M_\theta/P \) or \( M_\theta \cdot d/M_c \) or \( M_\theta \cdot d/M_L \)
- \( n_\theta = N_\theta \cdot T/P \) or \( N_\theta \cdot T \cdot d/M_c \) or \( N_\theta \cdot T \cdot d/M_L \)

where \( D \) is the mean diameter of the shell; \( d \) - the outside diameter of the nozzle; \( T \) - the thickness of the shell 1; \( t \) - the thickness of the nozzle; \( L \) - the length of the shell 1; \( t \) is \( P \) - the load on axial direction; \( V_c \), \( M_c \) - the loads on circumferential direction; \( V_L \), \( M_L \) - the loads on longitudinal direction; \( N_r, N_\theta \) - the membrane force per unit length of the cylindrical shell on radial and circumferential direction, respectively; \( M_r, M_\theta \) - the bending moments per unit length of cylindrical shell wall on radial and circumferential direction, respectively (Figure 5).

Normal stresses overlap by algebraic summation provided that they are within the linear – elastic range. It results the total normal stresses in:

- meridional direction of the shell 1,
  \[
  \sigma_j = \sigma_j(P) + \sigma_j(M_L)
  \]  
(16)

- circumferential direction of the shell 1,
  \[
  \sigma_j = \sigma_j(P) + \sigma_j(M_c)
  \]  
(17)

where \( j = r \) or \( \theta \).

In the same way, the tangential stresses are summed up algebraically:

\[
\tau = \tau(M_r) + \tau(V_c) + \tau(V_L)
\]  
(18)

3. CALCULATION METHODS FOR THE NOZZLE – SHELL INTERSECTIONS ACCORDING TO THE PRESENT CALCULATION NORMS

In norm [7] it is recommended that the local stresses in the joints attached to the shells subjected to external loads should be evaluated using the Calculation Bulletin WRC 107 [4], the WRC 297 [5] or by the finite element method. In code [8], the following papers are given as references for the design of the nozzle attached to the shells or to the branches: WRC 107 [4], WRC 297 [5], WRC 198 [6] and British Standard PD - 5500 [9].

3.1. Spherical shell - cylindrical nozzle joints

In the spirit of the norms PD 5500 [9] and EN 13445-3 [10], by local load one means a direct force, shear force, bending moment applied to a nozzle, an attachment whose action is different from that of the pressure in the vessel. The method refers to both the joints at the same level with the shells and to those protruding into the
vessel. The nozzles whose protrusion is lower than $\sqrt{2rt}$, will be treated as nozzles without protrusion. According to norms [9, 10] it is calculated the maximum stresses caused by the external load (internal pressure, axial force, bending moment and shear force) at the intersection of the joints with the shell through a general relation of the form,

$$\sigma_{\text{max}} = K(s) \cdot f(S)$$

(19)

where $K(s)$ is the concentration coefficient when applying the $S$ load, which can be internal pressure ($p = S$), axial force ($Q = S$), bending moment ($M = S$), and shear force ($V = S$).

The value of the concentration coefficient depends on the ratio $t/T$ between the wall nozzle thickness $t$ and the shell thickness $T$, and also on the geometrical simplex $\rho = r/R \sqrt{R/T'}$ where $R$ is the mean radius of the spherical shell, $T'$ is the local shell thickness, adjacent to the nozzle ($T < T'$) and $r$ is the mean radius of the nozzle (Figure 6). In general, $K(S)$ increases as the ratio $t/T$ decreases and $\rho$ increases. For the same values of the $t/T$ and $\rho$ the concentration coefficient is lower to the protrusion nozzles than to those attached on the shell (Figure 6). The function $f(S)$ form depends on the type of the $S$ load and shell geometry.

![Fig. 6. Cylindrical flush nozzle (a) and protruding nozzle (b) loads intersected with the spherical shell (2).](image)

In figure 6 there are listed the tasks for which the maximum stresses are calculated in the spherical shell - cylindrical nozzle intersecting section. The following expression are used for the maximal stresses [9]:

- internal pressure,

$$\sigma_{\text{max}} = K(p) \times \frac{p \cdot R}{2 \cdot T'}$$

(20)

- radial force $Q$ on the shell (axial in nozzle),

$$\sigma_{\text{max}} = K(Q) \times \frac{Q}{2\pi \cdot r \cdot T'} \sqrt{\frac{R}{T'}}$$

(21)

- bending moment,
\[ \sigma_{\text{max}} = K(M) \times \frac{M}{\pi \cdot r^2 \cdot T'} \sqrt{\frac{R}{T'}} \]  
\[ \sigma_{\text{max}} = K(V) \times \frac{V}{2\pi \cdot r \cdot T'} \]  
\[ \sigma_{\text{max}} = K(p) \times \frac{p}{\pi \cdot r \cdot T'} \]

where the following notations were used: \( M \) is the external moment applied to the nozzle; \( p \) is the internal pressure; \( Q \) is the radial load; \( V \) is the transversal force applied to the nozzle; \( \sigma_{\text{max}} \) is the maximum stress due to local loading; \( \sigma_\theta, \sigma_z \) is the circumferential and meridional stress, respectively; \( \sigma_y \) is the yield stress.

The calculation method of \([9, 10]\), in that it uses the stress concentration factors, practically leads to the doubling of the maximum stresses in the nozzle – shell intersection. In the case of loads superpositions (\( p, V \) and \( M \)) one has to observe the following condition,

\[ \frac{p}{p_0} + \frac{q}{q_0} + \frac{m}{m_0} \leq 1 \]  
\[ \bar{p} = \frac{p \cdot R}{2 \cdot T' \sigma_y} \]
\[ \bar{q} = \frac{1}{2\pi \cdot r} \cdot \frac{Q}{T' \cdot \sigma_y} \sqrt{\frac{R}{T'}} \]
\[ \bar{m} = \frac{M}{\pi \cdot r^2 \cdot T' \cdot \sigma_y} \sqrt{\frac{R}{T'}} \]  
\[ \frac{\sigma(p)}{\sigma_y} ; \frac{\sigma(V)}{\sigma_y} \sqrt{\frac{R}{T'}} ; \frac{\sigma(M)}{\sigma_y} \sqrt{\frac{R}{T'}} \]

With the notations of \([13]\), taking into account the principle of critical energy, it results:

\[ \bar{p} = \frac{\sigma(p)}{\sigma_y} ; \bar{q} = \frac{\sigma(V)}{\sigma_y} \sqrt{\frac{R}{T'}} ; \bar{m} = \frac{\sigma(M)}{\sigma_y} \sqrt{\frac{R}{T'}} \]  

where \( \bar{m} \) is the external moment shakedown factor; \( \bar{p} \) is the internal pressure shakedown factor; \( \bar{q} \) is the radial thrust shakedown factor; \( \bar{p}_0 \) is the overall internal pressure shakedown factor; \( \bar{q}_0 \) is the overall radial thrust shakedown factor; \( \bar{m}_0 \) is the overall external moment shakedown factor, dependent on the geometric

simplexes \( \rho = \frac{r}{R} \cdot \sqrt{\frac{R}{T'}} \) and \( t/T' \) or \( R/T' \) for \( t/T = \text{constant} \), which decreases with the increase of \( \rho \) and grow with increases of the ratio \( t/T' \).

3.2. Spherical / cylindrical shell – cylindrical nozzle junction

In what follows it is described an alternative calculation method in \([9, 10]\). The load components on the axes and reference system directions xyz are listed in figure 7.
a. Strength calculation.
It is calculated the values of the maximum allowable pressure \( p_{\text{max}} \), axial force \( F_{\text{max}} \) and bending moment \( M_{\text{max}} \) with specific normative relations, also considering, when appropriate, the contribution of the plate material compensation. In the case of the superposition of the pressure effects \( p \), of the axial force \( F \) and of the bending moment \( M_{\text{max}} \) on the nozzle the admissibility condition is:

\[
\frac{p}{p_{\text{max}}} + \frac{F_z}{F_{z,\text{max}}} + \frac{M_B}{M_{B,\text{max}}} \leq 1
\]  

(27)

where \( p_{\text{max}} \), \( F_{\text{max}} \) and \( M_{\text{max}} \) are the maximally admissible loads calculated by means of the relations presented in the norms. These have to meet the requirements:

\[
\frac{p}{p_{\text{max}}} \leq 1; \quad \frac{F_z}{F_{z,\text{max}}} \leq 1; \quad \frac{M_B}{M_{B,\text{max}}} \leq 1
\]

The nozzle is verified in terms of mechanical strength by means of the relation

\[
\sigma(p) + \sigma(M) + \sigma(F_z) \leq f_n
\]

which after successive replacements for the nozzle becomes,

\[
\frac{pd}{4e_{ab}} + \frac{4M_B}{\pi d^2 \cdot e_{ab}} + \frac{F_z}{\pi d \cdot e_{ab}} \leq f_n
\]  

(28)

where \( f_n \) is the allowable design stress of the nozzle material; \( d \) is the outer diameter of the nozzle; \( e_{ab} \) is the wall nozzle thickness. Since the stresses are algebraically summed, this means that it is allowed the linear elastic behavior of the nozzle material.

b. Shells buckling analysis with meridional stresses
The following conditions at the meridional stress have to be observed:
\[
\frac{M}{M_{\text{max}}} + \frac{|F_z|}{F_{z,\text{max}}} \leq 1 \quad (\text{if } F_z \text{ produces compression stress in the nozzle});
\]

\[
\frac{M}{M_{\text{max}}} \leq 1 \quad (\text{if } M \text{ produces compression stress in the nozzle}).
\]

In the previous relations

\[
\begin{align*}
M_{\text{max}} &= \pi \cdot r^2 \cdot t \cdot \sigma_{\text{max},c} \\
F_{\text{max}} &= 2 \cdot \pi \cdot r \cdot t \cdot \sigma_{\text{max},c},
\end{align*}
\]

(29)

where \( \sigma_{\text{max},c} \) is the maximum compression allowable stress in the nozzle. The relations are valid only in the case of the buckling in the linear – elastic field.

4. PROPOSAL ON THE APPLICATION OF THE CRITICAL ENERGY PRINCIPLE TO THE CALCULATION OF NOZZLE-SHELL JUNCTION

Problem solving of the stress state at the intersections of a nozzle and a shell can be done by using the critical energy principle set by V.V. Jinescu [13, 14]. By mixed – mode or composed load one means the simultaneous application of axial stress, bending stress, torsional stress. In [13-17], this type of application extends by adding normal stresses and shear stresses in the section. The method developed in [13, 15] is applied, generally, to nonlinear power law material behavior.

In the case of the shells intersection (Figure 7), the load is considered plane which requires the calculation of the load effects along the main directions: meridional \( Ox \) and circumferential \( Oy \) [16]. It is selected a system of axes \( Oxyz \) (Figure 7) with the origin at the opening center in shell 2. In order to do the strength calculations in the joint section, the shear force \( Q \) and the bending moment \( M \) are decomposed along the coordinating axes \( Ox \) and \( Oy \) thus (Figure 7):

\[
\begin{align*}
Q &= \sqrt{Q_x^2 + Q_y^2} \\
M &= \sqrt{M_x^2 + M_y^2}
\end{align*}
\]

(30)

The twisting moment (torque), represented as a vector, acts along the axis \( z \) so that \( M_z = M_{\theta} \). The total effect of these loads in the joint section between the two shells is obtained by the superposition of the effects. Currently, the calculation of stresses in the two shells is done considering the linear - elastic behavior of the materials. In the norms of calculation for such joints, exceeding of the yield strength up to twice the yield stress is admitted. In reality, within the interval \( \sigma_y \leq 2\sigma_y \), the material behaves nonlinearly - elastically, which requires reconsidering the current calculation methods, which refer to the materials with linear elastic behavior.

At any point on the joint circumference, it is calculated the circumferential \( \sigma_\theta \) and meridional \( \sigma_x \) stress, both on the inner and the outer shell (2) surface. These stresses result from loads superposition:

\[
\begin{align*}
\sigma_x &= \sigma_x(p_i; F_x; Q_x; M_x) \\
\sigma_\theta &= \sigma_\theta(p_i; F_x; Q_y; M_y) \\
\tau_\theta &= \tau_\theta(M_{\theta})
\end{align*}
\]

(31)
where \( p_i \) is the internal pressure in the vessel.

a. Load in the case of the linear - elastic behavior. We use algebraic summation of the stresses produced by each load:

\[
\begin{align*}
\sigma_x &= \sigma_x(p_i) + \sigma_x(F) + \sigma_x(Q_x) + \sigma_x(M_x) \\
\sigma_y &= \sigma_y(p_i) + \sigma_y(F) + \sigma_y(Q_y) + \sigma_y(M_y)
\end{align*}
\]  

(32)

For nonlinear - elastic behavior of the material, such an algebraic summation is not correct.

b. Load for materials with nonlinear behavior \([13 \div 17]\) consists in the participation calculation of the total specific energy of the actions applied to some structures of such material. For mechanical strain with normal \( \sigma \) and tangential \( \tau \) stress, the nonlinear, power laws, behavior is given by the eq. \([16]\):

\[
\begin{align*}
\sigma &= M_\sigma \cdot \varepsilon^k \\
\tau &= M_\tau \cdot \gamma^{k_1}
\end{align*}
\]  

(33)

where \( M_\sigma, M_\tau, k \) and \( k_1 \) are material constants; \( \varepsilon \) - strain; \( \gamma \) - shear strain.

For the mechanical behavior of the material structure given by relations (33), the following eq. of the total participation of the specific energies was obtained \([15, 18, 19]\),

\[
P_T = \left( \frac{\sigma}{\sigma_{cr}} \right)^{\alpha + 1} + \left( \frac{\sigma_b}{\sigma_{b,cr}} \right)^{\alpha + 1} \cdot \delta_b + \left( \frac{\tau}{\tau_{cr}} \right)^{\alpha_1 + 1} \cdot \delta_\tau + \left( \frac{\tau_{t}}{\tau_{t,cr}} \right)^{\alpha_1 + 1} \cdot \delta_{t}
\]  

(34)

where \( \sigma \) is the axial stress; \( \sigma_b \) is the bending stress, \( \tau \) is the shear stress, \( \tau_t \) is the twisting (torsion) stress, \( \alpha = 1/k \) and \( \alpha_1 = 1/k_1 \) and \( \delta_b, \delta_\tau \) are equal with 1 for loads producing compression and with -1 on the surfaces on which these produce stretching, \( \delta_{t} = 1 \).

In equation (34) the critical stresses are chosen according to the load type (force, bending moment, torque etc.). When evaluating the load application in relation to the permissible state, the equation (34) becomes:

\[
P_T^* = \left( \frac{\sigma}{\sigma_{al}} \right)^{\alpha + 1} + \left( \frac{\sigma_b}{\sigma_{b,al}} \right)^{\alpha + 1} \cdot \delta_b + \left( \frac{\tau}{\tau_{al}} \right)^{\alpha_1 + 1} \cdot \delta_\tau + \left( \frac{\tau_{t,al}}{\tau_{t,cr}} \right)^{\alpha_1 + 1} \cdot \delta_{t}
\]  

(35)

where at the denominators it was introduced allowable stresses \( \sigma_{al} = \sigma_{cr}/c_\sigma \), \( \sigma_{b,al} = \sigma_{b,cr}/c_{\sigma,b} \), \( \tau_{al} = \tau_{cr}/c_\tau \), \( \tau_{t,al} = \tau_{t,cr}/c_{\tau,t} \) and \( c_\sigma, c_\tau \) are the safety coefficients.

c. Bending and torsion loading.
For a nonlinear material behavior, when applying the load only to bending and torsion, the total participation is obtained from the equation (34) with \( \tau = \sigma = 0 \),

\[
P_T = \left( \frac{\sigma_b}{\sigma_{b,cr}} \right)^{\alpha + 1} + \left( \frac{\tau_{t}}{\tau_{t,cr}} \right)^{\alpha_1 + 1}
\]  

(36)
According to the law of equivalence of processes and phenomena [16, 17], if the same effect is produced by an
equivalent bending stress \( \sigma_{b,eq} \), then:

\[
P_T = \left( \frac{\sigma_{b,eq}}{\sigma_{b,cr}} \right)^{\alpha+1}
\]  

(37)

where \( \sigma_{b,eq} \) is the bending equivalent stress to the combined action of bending and torsion.

After equalizing the relations (36) and (37), it was obtained [16]:

\[
\sigma_{b,eq} = \left( \sigma_b^{\alpha+1} + K \cdot \tau_t^{\alpha_1+1} \right)^{\frac{1}{\alpha+1}}
\]  

(38)

where \( K = \frac{\sigma_{b,cr}^{\alpha+1}}{\tau_t^{\alpha_1+1}} \) represents a ratio characteristic to the material for the respective load application case.

A relation similar to (38), resulted from probabilistic considerations [20], in which, however, it was not taken
into account the behavior of the material, so that the exponents \((\alpha + 1)\) and \((\alpha_1 + 1)\) are not defined. This
proves that the inferences made on the basis of the critical energy principle are complete: the exponents of
equation (38) are well defined, they depend on the behavior of the material and on the load increase duration up
to its maximum value [13 - 15]. For a material with linear– elastic behavior \( \alpha = \alpha_1 = 1 \), thus:

\[
\sigma_{b,eq} = \left( \sigma_b^2 + K \cdot \tau_t^2 \right)^{\frac{1}{2}}
\]  

(39)

Assuming that the yield stress represents the critical state, then in the expression \( K \) will be replaced \( \sigma_{b,cr} \) with
\( \sigma_y \) and \( \tau_{t,cr} \) with \( \tau_y \), so that \( K = \left( \sigma_y / \tau_y \right)^2 \).

K’s value, as used in the Energonics method and how it is practically found, is a characteristic of the material,
which takes into account both the difference in bending and torsional behavior and the variation mode in time of
each load application (static or variable: pulsating, alternating symmetric etc.). K’s value should not be
correlated with the hypothesis of calculating equivalent stress [16].

5. CONCLUSIONS

The paper has analyzed the effect of the nozzle - shell intersection to the state of stresses, starting with the works
of Bijlaard. It presents the calculation particularities of stresses in the intersection area as they are stated in the
Welding Research Council Bulletin 107 and 297 and the ASME Code, Section VIII, British Standard PD - 5500
and EN 13445-3. It shows the limits, for the design practice, of each of these bibliographical references.

These can be applied only to the linear - elastic behavior of the materials of the components. In the case of
nonlinear - elastic behavior, it is shown how the problem can be solved using the concept of specific energy
participation introduced by the principle of critical energy. The calculation method with the consideration of the
stress concentration coefficient [9, 10] leads practically to the doubling of the maximum stresses calculated at the
cylindrical nozzle - spherical shell intersection.
The cumulation of the actions of these stabilizing factors for simultaneous load application is given by the amount in the left side of equation (24). Load application status is unacceptable if this amount is higher than the unity, in which case the nozzle – shell geometry is revised until the condition (24) is fulfilled.

The alternative methods for calculating local loads for the radial nozzle - spherical or cylindrical shells junction [9, 10] requires the calculation of each individual load (the pressure is relative to the permissible state, the force is relative to the maximum allowable nozzle force, the bending moment is relative to the maximum bending moment). For example: \( \Phi_p = \frac{p}{p_{\text{max}}} ; \Phi_Z = \frac{F_Z}{F_{Z,\text{max}}} ; \Phi_B = \frac{M_B}{M_{B,\text{max}}} \).

For all load application cases, that may appear in exploitation, the load application variations (\( \Delta p, \Delta F_Z, \Delta M_B \)) and the resulted stresses (\( \sigma_p, \sigma_{F_Z}, \sigma_{M_B} \)) are calculated. For using the capacity of the strain take-over, even when exceeding the yield stress, for materials with a relatively high capacity of deformation, it was admitted the following condition in the nozzle-shell joint section, unjustifiable in any way in the norm [9], \( \sigma_{\text{tot}} \leq 3 f_s \) where \( \sigma_{\text{tot}} = \sigma_T + \sigma_{\text{ech,mec}} \) is calculated for linear–elastic behavior; \( f_s \) is the allowable stress of the shell material; \( \sigma_T \) is the thermal stress due to the temperature differences through the wall thickness.

By accepting the non-linear behavior for \( \sigma > \sigma_y \), it is necessary to reconsider the current calculation methods (which relate to constructions from materials with nonlinear-elastic behavior, especially those caused by cracks and defects in the joint). The general case of the cracks influence on the deterioration and of this on the strength of the structure was treated in [20 - 23]. To solve the nozzle-shell junction problem completely, the damage influence has to be taken into consideration. In this paper, we made an initiation of the problem by pointing out how we can obtain the total effect in the nozzle- shell junction, using the Energonics method based on the concept of specific energy participation as shown in chapter 4 of this paper [13 - 16].

REFERENCES