DETERMINING DYNAMIC ACCURACY INDICATORS OF MULTICOORDINATE WORKING MACHINES IN THE FORM OF ROD STRUCTURES FOR FUZZY INERTIA AND DISSIPATION PARAMETERS

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Abstract: The paper considers a manipulator design and its dynamic model developed by the authors. On the basis of this dynamic model a mathematical model of the manipulator has been created, which enables its spatial motion investigation. The mathematical model takes into account the manipulator inertia tensor with fuzzy components. The developed computation procedure makes it possible to determine cross-angular displacements of the manipulator for impact and sinusoidal laws of torque variation. During random variations of the moment of external forces, complex cross-angular displacements of the manipulator occur. The displacement trajectory includes an elliptical region that corresponds to the ellipse of the manipulator elastic system stiffness. The range of the manipulator upper-end displacement trajectory is maximal in the direction close to that of minimal cross-angular stiffness of the manipulator elastic system.

Keywords: manipulator, spatial motion, inertia tensor, dynamic model, manipulator cross-angular displacements

1. INTRODUCTION

Advanced designs of working machines of the manipulator type are built on the basis of rod structures. Basic parameters of these machines are speed of response and positioning accuracy. Therefore, it is important to investigate dynamic properties of such machines.

In general terms, the problem consists in improving dynamic characteristics of working machines in the form of rod structures.

This problem is connected with important scientific and practical tasks related to creation and application of working machines. Improvement of their dynamic performance is the basis for developing advanced designs of manipulators, construction and road machines.

Recent studies and publications pay considerable attention to the problem of determining static and dynamic characteristics of the working machines based on manipulators [1].

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of working machines have been investigated in [2]. A number of publications are directed towards studying
kinematics of working machines [3]. In the literature main attention is paid to the problems of statics [4]. Some
publications [5] contain research on peculiarities of the working machine spatial motion. Dynamic vibration
processes are shown to be important in this respect [4]. For studying spatial motion, the mechanism inertia
parameters are used in the form of the inertia moment tensor [6].

In the literary sources there were found no results on studying dynamic accuracy indicators of working
machines, which take into account fuzzy inertia and dissipation parameters.

Unsolved aspects of the general problem include determination of the dynamic accuracy indicators for
multicoordinate working machines and their investigation.

The research aims at the development of a method for determining dynamic accuracy of multi-coordinate
working machines in the form of rod structures. Research tasks include analysis of the working machine design
solution, elaboration of the dynamic and mathematical models of the working machine dynamics, algorithmic
and software provision to be used as a basis for determining dynamic accuracy indicators, particularly, the
positioning accuracy.

2. PRESENTATION OF THE MAIN RESEARCH MATERIAL

2.1 Manipulator of a multi-coordinate working machine, its dynamic and mathematical models

Manipulator, mounted on a rotary base, is the basis of the developed working machine. Manipulator includes
pillar 1 where an articulated rod structure with drives is located (Figure 1).

![Fig. 1. General view of the developed manipulator of the working machine: 1 – pillar; 2 – boom; 3 – drive; 4 – articulated joints; 5 – actuator.](image)

Manipulator is a spatial rod structure with articulated joints. A considerable number of extended elements (rods)
2 with loose joints 4 determine uncertainty of the dynamic system dissipation parameters and variations of its
inertia characteristics.

Geometrical parameters of the manipulator are chosen so that they satisfy the condition of providing the required
displacements of working member 5. It should be taken into account that in the process of manipulator motion
significant accelerations and, therefore, intensive inertia loads are observed.
During manipulator positioning, spatial vibrational displacements of the working member occur. Main indicators of the dynamic positioning accuracy are the character of actuator dynamic displacements, their frequency and vibration modes. An essential parameter is the time of the transient process during actuator positioning.

Manipulator of a working machine is a complex dynamic system that includes rods 1 – 5 with articulated joints. (Figure 2). In the manipulator operation process, its rotational motion relative to the center of the main mounting assembly O takes place (Figure 2).

The manipulator comprises several massive series-connected components in the form of rods 1…5. Under the action of the drives and dynamic forces, mutual location of the components is changed as well as characteristics of the articulated joints. During manipulator positioning, spatial motion of the load 6 is provided.

Due to the considerable length of the manipulator during positioning of the load, its close-to-spherical motion relative to the center of mounting assembly O occurs.

Manipulator is a complex rod structure with articulated joints. Each joint has respective dissipation characteristics (friction forces) as well as contact stiffness, plays and clearances.

When the manipulator is operated, changes in the mutual location of the rods, forming a kinematic chain, are observed. This leads to the changes of the system dynamic properties when it rotates around the articulated mounting assembly O. Inertia properties of the manipulator are given in the form of fuzzy inertia parameters represented by the system inertia tensor components. Articulated joint O is assumed to be a point with elastic-dissipative links along cross-angular displacements. In order to determine the motion of the load during its positioning, equation of the spherical motion of a solid body with inertia properties is used. A corresponding dynamic model has been elaborated (Figure 3).
In accordance with the dynamic model, equations of the manipulator spherical motion have been elaborated. For a general case, equation of spherical motion of the manipulator as a solid body [7] during its positioning is given by:

$$\frac{d\vec{L}}{dt} + \vec{\omega} \times \vec{L} = \vec{M},$$

where: $\vec{L}$ is vector of the manipulator kinematic momentum with a fixed point; $\vec{\omega}$ – vector of the manipulator angular velocity; $\vec{M}$ – vector of the resultant moment acting on the manipulator.

Kinematic momentum equals the product of the manipulator inertia tensor in the chosen coordinate system $x$, $y$, $z$ multiplied by the angular velocity vector:

$$\vec{L} = I \cdot \vec{\omega}.$$  (1)

In projection on the coordinate axes, equations of the manipulator spherical motion (1) have the form of:

$$\frac{dL_x}{dt} + \omega_y L_z - \omega_z L_y = M_x,$$

$$\frac{dL_y}{dt} + \omega_z L_x - \omega_x L_z = M_y,$$

$$\frac{dL_z}{dt} + \omega_x L_y - \omega_y L_x = M_z,$$  (2)

where: $M_x, M_y, M_z$ is projections of the resultant moment of external forces, acting on the manipulator, relative to point $O$, on coordinate axes $x$, $y$, $z$; $\omega_x, \omega_y, \omega_z$ – projections of the solid body angular velocity on coordinate axes $x$, $y$, $z$.

The moments are the sum of the useful load moments determined by the action of dynamic force $\vec{P}$ and the moments of deformation and dissipation forces. Moments of the forces relative to the axes are determined by the dependencies.

$$M_x = P_x \cdot \tau + c_\psi \psi + b_\psi \omega_z,$$

$$M_y = P_y \cdot \tau + c_\theta \theta + b_\theta \omega_y,$$

$$M_z = P_z \cdot \tau + c_\varphi \varphi + b_\varphi \omega_z,$$  (3)

Where: $P_x, P_y, P_z$ are dynamic load projections on the actuator, $l$ – distance from the point of force application to the center of mounting assembly $O$, $c_\psi, c_\theta, c_\varphi$ – components of the lateral angular stiffness of the manipulator during its rotation about the center; $b_\psi, b_\theta, b_\varphi$ – coefficients of the manipulator resistance about axes $x$, $y$, $z$; $\psi, \theta, \varphi$ – Euler – Krylov angles [7, 8] that determine spatial cross-angular displacements of the manipulator.

Equation system (2) of the manipulator spherical motion includes complexes $L_x, L_y, L_z$, which determine projections of the manipulator angular momentum:
\[ L_x = I_{xx} \omega_x - I_{xy} \omega_y - I_{xz} \omega_z; \]
\[ L_y = -I_{xy} \omega_x + I_{yy} \omega_y - I_{yz} \omega_z; \]
\[ L_z = -I_{xz} \omega_x - I_{yz} \omega_y + I_{zz} \omega_z. \]  

(4)

Coefficients of formulas (4) are the manipulator inertia tensor components:

\[
(I_y) = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}.
\]

(5)

Inertia tensor is symmetrical and, therefore, \( I_{yx} = I_{xy}, I_{yz} = I_{zy}, I_{zx} = I_{xz}. \)

Moment-of-inertia tensor is calculated using a special procedure [9] that takes into account changes of its components as fuzzy sets. Equations of spherical motion were used for the development of a numerical calculation procedure. For this, differential equation system (2) was reduced to the system of three integral equations. From them, projections of the kinematic momentum were determined:

\[
L_x = L_{x0} + \int_0^t (M_x - \omega_y L_y + \omega_z L_z) dt, \\
L_y = L_{y0} + \int_0^t (M_y - \omega_z L_z + \omega_x L_x) dt, \\
L_z = L_{z0} + \int_0^t (M_z - \omega_x L_x + \omega_y L_y) dt,
\]

(6)

where \( L_{x0}, L_{y0}, L_{z0} \) are initial values of the kinematic momentum projection.

Values in the right-hand side of formulas (6) and (3) are input parameters of the calculation procedure, and parameters in the left-hand sides of the formulas are the output parameters. Input and output parameters form an open-loop cause-effect relationship. This relationship is realized in the general mathematical model.

The above calculation procedure is presented in the form of a unit, input parameters of which are projections \( M_x, M_y, M_z \) of the manipulator external load moment; projections \( \omega_x, \omega_y, \omega_z \) of the manipulator angular velocity and initial (input) values of \( L_x, L_y, L_z \) of the kinematic momentum projections [7].

Angular velocity vector is calculated by a sequential forward calculation procedure. For this dependency (4), transformed to the following form, are used:

\[
\omega_x = \frac{L_x + I_{xx} \omega_x + I_{xy} \omega_y}{I_{xx}}, \\
\omega_y = \frac{L_y + I_{xy} \omega_x + I_{yy} \omega_y}{I_{yy}}, \\
\omega_z = \frac{L_z + I_{xz} \omega_x + I_{zz} \omega_z}{I_{zz}}.
\]

(7)
Dependencies (7) are realized in a special unit.

The following parameters serve as the input parameters of the unit: kinematic momentum vector $\mathbf{L}$ calculated by formula (6), angular velocity vector $\mathbf{\omega}$ and components of the inertia tensor $(I_{ij})$. The calculated new values of the angular velocity vector are the output of the unit.

Dynamic moment acting on the system is calculated using a special procedure in the block model that realizes functional dependencies (3). The moment depends on the vector of the manipulator angular position and angular velocity vector in accordance with formulas (3).

The moment acting on the manipulator in the presence of elastic-dissipative links has been calculated. Each of the moment components (projections) is computed by a separate unit included into the general structure of the block model. The presence of resistance moments, connected with elastic and dissipative links of the manipulator, is taken into account.

The units are combined into a singular structural model of the system [10]. It is intended for calculating the process of the manipulator cross-angular displacements under the action of the moment with projections $M_x, M_y, M_z$. The model also includes means for displaying modeling results (Figure 4).

3. RESULTS OF THE MATHEMATICAL MODELING OF THE MANIPULATOR CROSS-ANGULAR DISPLACEMENTS

The developed mathematical model was used for calculating the spatial cross-angular displacements of the manipulator. Modeling was performed for different variation laws of external torque acting on the manipulator. Impact (pulse) variations of the torque as well as sinusoidal and random torque variations are considered. This corresponds to the dynamic loads of the working machine.
Characteristic is manipulator displacement under impact (pulse) torque variation. Pulse variation of the external momentum load causes complex cross-angular manipulator vibrations.

Cross-angular displacements of the manipulator in two directions reflect a complex trajectory of the manipulator actuator motion. Typical trajectory of cross-angular displacements under pulse load have a number of sequential loop-shaped patterns, the direction of which is interconnected with the directions of main axes of the manipulator elastic system stiffness (Figure 5).

From calculation results, it follows that the actuator displacement trajectories are determined by pulse loads. Maximal amplitude of the loop-shaped displacement is achieved in the direction of the axis of elastic system minimal stiffness. Loop-shaped displacements occur periodically with gradual reduction of the amplitude and changes in the phase of the loop-shaped trajectory location. Ultimately, vibrations are damped and the manipulator returns in its initial (zero) position.

External torque loads on the manipulator are, as a rule, limited and smoothly variable. A typical torque load of such type is a sinusoidal torque variation. During sinusoidal variations of the manipulator torque load its dynamic displacements, after a certain transient process, acquire a harmonic (sinusoidal) character. Transient processes are damped after 2…3 periods of oscillations. After that steady harmonic cross-angular displacements of the manipulator are observed. Displacement trajectories acquire an elliptical form with axes oriented along the main axes of the manipulator elastic system stiffness (Figure 6).

Fig. 5. Trajectories of the manipulator actuator displacements under pulse variations of the manipulator external load torque.

Fig. 6. Actuator displacement trajectories for sinusoidal variations of the torque acting on the manipulator.
4. CONCLUSIONS

1. It has been determined that for mathematical modeling of the manipulator spatial cross-angular displacements a mathematical model, corresponding to spherical motion of a solid body, is expedient to be used. To make the procedure of mathematical modeling of the manipulator spherical motion more reliable, spherical motion equation system was reduced to the form of the integral equation system with input parameters defined by the introduction of special feedbacks.

2. Various external loads act on the manipulator dynamic system. Typical among them are impact (pulse) loads, sinusoidal loads and broadband random torque variations. Under random changes of the torque of external forces, acting on the manipulator, its complex cross-angular displacements occur. Displacement trajectories cover an elliptical region that, on the whole, corresponds to the ellipse of the manipulator elastic system stiffness. The range of the displacement trajectory of manipulator upper end is within 6 mm and is maximal in the direction close to that of minimal cross-angular stiffness of the manipulator elastic system.

3. As further direction of the research, it is recommended to take into account changes in the inertia moment tensor, i.e. to take into account changes in the tensor field of inertia moment tensor of the manipulator dynamic system, caused by spatial cross-angular displacements of the manipulator relative to the absolute coordinate system.

REFERENCES