NEW PORTFOLIO RISK OPTIMISATION METHOD FOR STRONGLY DEPENDENT ASSETS

PIOTR FRYDRYCH*1, ROMAN SZEWCZYK1

1Warsaw University of Technology, Institute of Metrology and Biomedical Engineering, Poland

Abstract: New market time series Multiplexed Hysteretic Threshold Autoregressive (MHTAR) model was developed to test the correlation between assets stability, which is the basis of MPT theory and other portfolio optimisation methods. New approach to risk optimisation was presented, which can efficiently lower risk for strongly dependent assets. Developed Random Trading Signals Multiplexing (RTSM) method enables diversification of risk for any portfolio and increase safety for assets with low liquidity.

Keywords: risk management, hysteresis model, correlation, market modelling

1. INTRODUCTION

Last Global Financial Crisis 2007-2011 has shown that inadequate risk estimation and optimisation methods may cause a total collapse of many large financial institutions. Most of them still use MPT (Modern Portfolio Theory) or newer methods based on similar assumptions to optimise their asset portfolio risk. The basis of MPT is independent random behaviour of the stock market. In the real market, stock returns are strongly dependent. That may cause important differences between theoretical risk values and its actual value. That shows a strong need for design of adequate portfolio risk estimation and optimisation methods that would be able to include the relationship between different investor decisions and interactions existing on the market. On the other hand, measurements of dependency strength between assets and prediction of chaotic behaviour are almost impossible. Existing methods are able only to poorly estimates some parameters for a short period of time. That implies a different approach to portfolio risk optimisation, which is presented in this paper.

2. RISK ESTIMATION METHODS

One of commonly used portfolio risk estimation method is VaR (Value at Risk) calculation developed by J. P. Morgan in 1994. It estimates maximum losses for given probability level in the time period, equation (1):

\[ P(V \leq V_0 - \text{VAR}) = \alpha \]  

(1)

where \( V_0 \) – present value, \( V \) – value at the end of the time period, \( \alpha \) – tolerance level (accepted loss probability)

VaR can be also described by return equations (2) and (3):

\[ P(R \leq R_{\alpha}) = \alpha \]  

(2)

\[ \text{VaR} = -R_{\alpha}V_0 \]  

(3)

* Corresponding author, email: piotr.frydrych@gmail.com
© 2014 Alma Mater Publishing House
where $R$ – rate value, $R_a$ – rate for tolerance level.

This basic idea enables risk assessment and comparison of different assets and portfolios. Presented approach of risk measurement seems to be useful and easily applicable, but practical estimation brings many problems.

Every VaR estimation method is a compromise between safety and openness for earning opportunities. Basel Committee recommends following exception rates (Table 1) [1].

<table>
<thead>
<tr>
<th>VaR confidence level</th>
<th>Non-rejection region for number of exceptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>255 days</td>
<td>510 days</td>
</tr>
<tr>
<td>99%</td>
<td>4&lt;N&lt;17</td>
</tr>
<tr>
<td>95%</td>
<td>37&lt;N&lt;65</td>
</tr>
<tr>
<td>90%</td>
<td>81&lt;N&lt;120</td>
</tr>
</tbody>
</table>

Efficient VaR estimation method should have appropriate exceptions level. Too cautious approach can prevent profitable investment opportunities.

Simplest method for VaR estimation is variance-covariance method used by Risk Metrics. It is based on assumption that stock returns have a normal distribution, equation (4):

\[
R_a = \mu - k\sigma
\]

Where $\mu$ - mean rate value, $k$ – tolerance factor, for $\alpha=0.05$, $k=1.65$, $\sigma$ – standard deviation.

Due to different shape of real stock rates distribution probability of return lower than $R_a$ can be higher than for a normal distribution. Many methods such Normal Inverse Gaussian and Generalised Hyperbolic distributions or Cornish-Fisher approximation [2] enable the estimation of real return distribution more properly. VaR calculation period selection has an important influence on return distribution shape because of long-term trends existence. To solve this problem, a longer period can be analysed and rescaled according for the actual number of days. It should be emphasised that for non-normal distributions rescaling factor is not equal to $\sqrt{t}$. Scaling index can be estimated using linear regression of logarithm of standard deviation, equations (5) and (6).

\[
\sigma = ct^\alpha
\]

\[
\ln(\sigma) = a\ln(t) + \ln(c)
\]

where $\sigma$ – standard deviation, $\alpha$ – scaling index, $t$ – time period, $c$ – some constant value.

Index estimation also gives information about dependency between neighbouring values. For $\alpha=0.5$ there is no dependence, for $\alpha>0.5$ dependency is positive, and for $\alpha<0.5$ negative. Index has a value less than 0.5 for market returns (Figure 1). This is because returns are connected with differences between day prices. Differences tend to zero for positive dependence of prices.

Rescaling distribution from a longer period of time results in higher variance in most cases. That cause higher VaR value, which can reduce investment ability. Method based on prediction can be an answer to this problem. Basis of such approach is assumption that there is the time dependence between neighbouring values of volatility and return.

Common model for this dependence estimation is GARCH (Generalized Auto-Regressive Conditional Heteroskedasticity) model. Research show its low exception rate and lower capital requirements [2]. The objection to this method is validation of the model. What is questionable, is assumption that there is only a time dependence in market time series.
3. PORTFOLIO RISK AND DIVERSIFICATION

VaR value for the portfolio in variance-covariance method is calculated as sum of assets variances and their covariance, equations (7-10):

\[
V = (\mu - k\sigma)V_0
\]

\[
\mu = \sum_{i=1}^{m} \mu_i
\]

\[
\sigma = \sum_{i=1}^{m} \sum_{j=1}^{m} w_i w_j \sigma_{ij}
\]

\[
\sigma_{ij} = \sigma_i + \sigma_j + \text{cov}_{ij}
\]

where \( \mu_i \) – asset return mean vale, \( w_i, w_j \) – partition weights, \( \sigma_{ij} \) – variance for portfolio, \( \text{cov}_{ij} \) – covariance.

Consequence of using these equations is that VaR depends strongly on correlation between assets. Markowitz proved that variance of the portfolio can be decreased by selection of portfolio with low correlation value. Estimation of correlation for non-normal distributed signals is problematic. It can be solved by Kendall [3] or Spearman rank [4] coefficients. Nonetheless, calculation problem is not the only one. Important question is, what is the probability of constant correlation between predicted period? What if correlation changes from low values to 0.9? It can dramatically change portfolio variance, as it is shown in previous equations.

4. DEVELOPED MULTIPLEXED HYSTERETIC THRESHOLD AUTOREGRESSIVE MODEL

To study the correlation instability, market time series model was developed. Purpose for a new model was to include not only dependence between neighbouring volatility and return values such it is in GARCH model, but also long term system memory. Threshold Vector Autoregressive (TVAR) and Self-exciting Threshold Autoregressive (SETAR) models, give a better approximation of nonlinear correlations and value dependencies on the market [5]. Developed Multiplexed Hysteretic Threshold Autoregressive (MHTAR) model was inspired by magnetic hysteresis model [6, 7]. It assumes that for each investor last return values and prices are less important than price that they paid for a specific asset. That causes hysteresis effect and strong crashes. That is parallel to magnetic materials that the internal structure is composition of interacting domains. Domain is a collection of particles oriented in the same direction. During magnetic field changes and its fluctuation in material particles changes its magnetisation as a group, not separately. It can be observed in trends in Barkhausen noise signal.

Let consider Markov process, equation (11):

\[
p_{i,j} = P(X_{n+1} = j | X_n = i)
\]
Dependence between neighbouring states is less than 0.5, equation (12), as it can be observed for S&P (Figure 1).

\[ P(X_{n+1} > 0|X_n > 0) < 0.5 \text{ and } P(X_{n+1} < 0|X_n < 0) < 0.5 \]  

(12)

Price relates also to buy and sell price. Probability of stop loss and profit occurrence is proportional to the volume of buy \( V_B \) or \( V_S \) sell transactions. Number of transactions is limited by maximum volume, which grows in time. Queued transactions are waiting for next day, what cause commutation and crash.

Stop loss for long position, equation (13):

\[ P(X_{n+1} < 0|\text{price}_n < \text{price}_B - \text{stop loss}) \approx V_B \]  

(13)

Stop loss for short position, equation (14):

\[ P(X_{n+1} > 0|\text{price}_n > \text{price}_S + \text{stop loss}) \approx V_S \]  

(14)

Profit for long position, equation (15):

\[ P(X_{n+1} > 0|\text{price}_n > \text{price}_B + \text{profit}) \approx V_B \]  

(15)

Profit for short position, equation (16):

\[ P(X_{n+1} < 0|\text{price}_n < \text{price}_S - \text{profit}) \approx V_S \]  

(16)

Dynamics of time series generated by this model have similar characteristic to real S&P index (Figure 2).

![Fig. 2. Simulation of market time series using MHTAR model.](image)

Distributions shapes for S&P and MHTAR model are similar (Figure 3).

Developed model exhibits important feature. It has strong deterministic relation which is hardly measurable using methods like autocorrelation. This is because of value dependence, which as opposed to time-dependence cannot be observed in basic relation between neighbouring values.
5. UNSTABLE ASSET CORRELATION RISK

That also causes strong uncertainty in the correlation between different assets. To test the influence of value dependence on the correlation, twenty independent time series were generated using presented model. Correlation was measured using Pearson’s $R^2$ coefficient. Correlation between independent time series vary from 0.4 to more than 0.9 (Figure 4). That shows, that correlation should be regarded as a random variable. It can be also proven by market experience. Even assets, which are correlated according to fundamental analysis in some periods seem to have a negative correlation. Dependency between S&P index and minor European indices are example of this phenomenon. It is not only metrological problem, but feature of the market. Unstable correlation between portfolio assets is important risk factor. Typical portfolio diversification methods are not able to overcome this problem sufficiently. Modern Portfolio Theory (MPT) and other methods are, therefore, applicable only for short periods of time, for which it can be assumed that correlation between assets is constant.

6. ILLIQUIDITY RISK

Value of risk calculation methods measures risks for the time period, e.g. daily risk or monthly risk. Real funds have to take also into account liquidity of the market. Often one month is not enough to close position during decreasing trend. In such cases, risk is higher than for monthly VaR [8]. Diversification of assets does not prevent
this problem. Even 1% of the whole portfolio can be an important part of shares for one asset or market in case of commodity exchange. Closing position can also deeper crash, which should be also considered in case of the higher percentage of shares. For that reason efficient portfolio risk optimisation method, be able to decrease the risk of illiquidity.

7. NEW APPROACH TO PORTFOLIO RISK OPTIMISATION

Efficient risk optimisation method should have following features:
- Enables profitable investment strategies;
- Decrease VaR;
- Decrease illiquidity risk;
- Applicable for different assets and markets.

In previously presented research, it was proven that dependency between assets can change in time. For this reason, it was assumed the worst case of portfolio with only one asset, which is equal to assets with maximum correlation. Random Trading Signals Multiplexing (RTSM) method is not based on statistical parameters of the asset, what makes it independent to market dynamics.

Idea of the new method is to diversify trading strategies and improve those statistics according to risk regulations. Let's consider trading strategy as buying $C_B$, equation (17) and selling conditions $C_S$, equation (18), which are time dependent variables for each asset.

\[
C_B(t) = \begin{cases} 
1 & \text{buy} \\
0 & \text{sell} 
\end{cases}
\]  

\[
C_S(t) = \begin{cases} 
1 & \text{sell} \\
0 & \text{buy} 
\end{cases}
\]  

In fact, VaR can be changed by $C_B(t)$, $C_S(t)$ and position value. It is not possible to change other factors. Investment time period depends also on strategy and market dynamics. It is possible to decrease VaR, by decreasing position value. If they would be opened using many independent strategies VaR of portfolio would be sum of positions which can be closed at the same time, equation (19).

\[
VaR_{opt} = VaR + \sum_{n=1}^{N} p(n)ndn
\]  

where $VaR_{opt}$ – optimised risk; $VaR$ – value at risk calculated for portfolio; $p(n)$ – probability of number of positions closed at the same time; $n$ – number of positions, $N$ – number of positions in portfolio.

In this research 1000 independent strategies with uniformly distributed random buying and selling conditions were tested on S&P index. Plot of change portfolio value in time shows that growth is more stable than for one strategy (Figure 5).
To show more clearly the probability of number of positions opened at the same time, a plot was made for only 50 independent strategies (Figure 6). It proves that developed method lowers portfolio risk efficiently. Probability is highest for one sale transaction at the same time.

![Fig. 6. Frequency of number of signals occurred at the same time.](image)

Optimised risk calculated using equation (19) can be strongly reduced by increasing the number of independent strategies (Figure 7). It should be emphasised that optimised VaR values were calculated for one asset. Buy and sell signals from strategies occur mostly not at the same time what lowers illiquidity risk.

![Fig. 7. Dependency of VaR decrease for portfolio on number of independent strategies.](image)

8. CONCLUSIONS

Presented research showed that commonly used risk calculation methods underestimates risk, what can be dangerous for financial institutions. On the other hand, more secure approach to risk calculation can prevent many profitable investment possibilities. Typical risk optimisation methods are focused on measuring and prediction few market features, such as return and volatility. Those parameters give only partial view on the whole market risk. Very often illiquidity risk is not included, and efficient methods for preventing it are not applied.
Developed value dependent model simulate dependencies existing on the market, which are hardly measurable with typical methods, thus their influence can be underestimated. It also shows that for processes with long-term trends like stock market time series correlation can change in time. That undermine most of the portfolio optimisation methods based on measurements of dependence between assets.

Proposed method for portfolio risk optimisation for highly dependent assets do not limit profits and does not increase risk capital regiments. It lowers VaR for portfolio and illiquidity risk efficiently. Comparing to typical approach it is not focused on market dynamics, but on those parameters of trading which can be improved. That enables application on many markets. Presented strategies are very simple, but further research can lead to improvement of methods with random parameters fluctuations.

REFERENCES