SIMULATING PASSIVE SUSPENSION ON AN UNEVEN TRACK SURFACE

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Abstract: Automobile suspension systems have an important role in a vehicle's functioning, especially with regard to driving safety. In the present paper we exhibit the equations that characterize a passive suspension system. Considering that solving the equations is extremely cumbersome we developed a simulation scheme in MATLAB Simulink. The simulation allows for an analysis of the behavior of the passive suspension system on any uneven track surface whose configuration is ensured by stimulus signals. For the simulation we used the quarter car model. The suspension was chosen as having two degrees of freedom.

Keywords: passive suspension systems, quarter car model, uneven track surface

1. INTRODUCTION

Automobile suspension systems are different from one to another depending on the manufacturer [1], a fact which gives the way for a great diversity between the models existing on the market. Regardless the solution chosen in the design phase, it is considered essential that, while functioning, the suspension system ensures vehicle safety. In addition, the suspension system must take over any irregularity of the track. It is known that the unevenness of the track produce oscillations in the vehicle's wheels [2, 3], oscillations that will be transferred to the wheels' arms. It becomes obvious the fact that the suspensions' role, as the intermediary between the wheel arms and the chassis, is to reduce as much as possible the vibrations and the shocks that appear during operation [4, 5]. It thus appears the necessity to use a suspension having a quality as high as possible.

Until the present date, passive, semi-active and active suspension systems have been developed. Passive systems are the normal variety [4]. Semi-active systems and active systems were introduced in order to improve suspensions parameters from a comfort point of view and to ensure a better adaptability to the track conditions [4]. Semi-active systems [5] use in general, magneto rheological dampers which can be controlled by PID, PI or PD controllers. Active suspension systems benefit from a hydraulic engine which inserts, by means of the system pressure, an additional force that can regulate the dampers irrespective of the forces applied by the track and chassis [4]. Although extremely promising, from a performance perspective, the semi-active and active systems are generally targeted at luxury automobiles or other vehicles with special purposes.

In order to ensure traveling comfort, it is necessary to isolate the body of the automobile, also called sprung mass, from the track irregularities [6]. It is also recommended [6] to decrease the frequency of the sprung mass down to values close to 1 Hz (a value which is known as the sensitive frequency of the human body) and to limit the frequency peak to a maximum of 10 Hz. It is necessary to study how a passive suspension system behaves when the track is uneven.

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2. CONSIDERENTS REGARDING THE MATHEMATICAL MODELLING OF A PASIVE SUSPENSION SYSTEM

We will take into consideration a vehicle suspension system, quarter car model, for which we will consider that the irregularities in the track surface have a direct action on the not suspended and suspended mass of the vehicle and on the passenger seat. The suspension system must support the vehicle and maintain the direction applied to the vehicle during maneuvers. Regardless of the track condition or the vehicle maneuvers, a new criteria must be regarded, specifically the passenger comfortableness. In order to solve these goals various constructive solutions were regarded, the more frequently adopted being the solution which involves using the spring and damper in parallel. Using this solution, it is ensured that the energy is stored, by means of the spring, and that it is also dissipated, by means of the damper. On passive systems, such as the ones analyzed in the present paper, the constructive parameters for the spring and damper remain unchanged. Unlike the semi-active and active suspension systems, for which the damper parameters can be changed, on passive systems it is important to choose the suspension elements depending on the tracks that will be used. Once the springs and the dampers are chosen, the suspension behavior cannot be altered, except as a result of extended usage. In Figure 1 it is presented a passive suspension system [7] quarter car model.

![Passive Suspension System](image)

In Figure 1 the meanings of the passive suspension system masses are: \( M_{se} \) - seat and driver mass, \( M_s \) - the quarter of the vehicle sprung mass, \( M_u \) - the quarter of the vehicle unsprung mass. These parameters are chosen depending on the type of vehicle for which the simulation is made. For the suspension elements (Figure 1) we take into consideration the coefficients: \( k_{se} \) - seat suspension spring stiffness, \( k_s \) - vehicle suspension spring stiffness, \( k_t \) - tire stiffness, \( b_{se} \) - damping ratio of the seat suspension, \( b_s \) - damping ratio of the vehicle suspension, \( Z_r \) - road input and \( Z_{se}, Z_s, Z_u \) - the arrow (vertical displacement) for driver and seat, for sprung mass, respectively for unspring mass.

The equations of the passive suspension system [7] that was presented in Figure 1 can be written for each mass in particular. Equation (1) is for drive and seat mass:

\[
M_{se} \frac{d^2 Z_{se}}{dt^2} + b_{se} \left( \frac{dZ_{se}}{dt} - \frac{dZ_s}{dt} \right) + k_{se} (Z_{se} - Z_s) = 0
\]  

which leads to equation:
\[
\frac{d^2 Z_{sc}}{dt^2} = -\frac{b_{se}}{M_{se}} \left( \frac{dZ_{sc}}{dt} - \frac{dZ_s}{dt} \right) - \frac{k_{se}}{M_{se}} \cdot (Z_{sc} - Z_s) = 0 \quad (2)
\]

For sprung mass, the obtained equation for the passive suspension system will be:

\[
M_s \cdot \frac{d^2 Z_{s}}{dt^2} - b_{se} \left( \frac{dZ_{sc}}{dt} - \frac{dZ_s}{dt} \right) + b_s \left( \frac{dZ_s}{dt} - \frac{dZ_u}{dt} \right) - k_{se} \cdot (Z_{sc} - Z_s) + k_s \cdot (Z_s - Z_u) = 0 \quad (3)
\]

As in the case above, using equation (3) we obtain:

\[
\frac{d^2 Z_{s}}{dt^2} = \frac{b_{se}}{M_s} \left( \frac{dZ_{sc}}{dt} - \frac{dZ_s}{dt} \right) - \frac{b_s}{M_s} \left( \frac{dZ_s}{dt} - \frac{dZ_u}{dt} \right) + \frac{k_{se}}{M_s} \cdot (Z_{sc} - Z_s) - \frac{k_s}{M_s} \cdot (Z_s - Z_u) = 0 \quad (4)
\]

For unsprung mass, the equation is:

\[
M_u \cdot \frac{d^2 Z_{u}}{dt^2} - b_s \left( \frac{dZ_s}{dt} - \frac{dZ_u}{dt} \right) - k_s \cdot (Z_s - Z_u) + k_t \cdot (Z_u - Z_f) = 0 \quad (5)
\]

A relation from which we obtain:

\[
\frac{d^2 Z_{u}}{dt^2} = \frac{b_s}{M_u} \left( \frac{dZ_s}{dt} - \frac{dZ_u}{dt} \right) + \frac{k_s}{M_u} \cdot (Z_s - Z_u) - \frac{k_t}{M_u} \cdot (Z_u - Z_f) = 0 \quad (6)
\]

Starting from equations (2), (4) and (6) it is suggested by [7] the state space variables:

\[
x_1 = \frac{dZ_{sc}}{dt}; \ x_2 = Z_s - Z_{sc}; \ x_3 = \frac{dZ_s}{dt}; \ x_4 = Z_u - Z_s; \ x_5 = \frac{dZ_u}{dt}; \ x_6 = Z_f - Z_u \quad (7)
\]

If we replace the state space variables in equations (2), (4) and (6) then, according to [7] the state space equations will be written in the form of:

\[
\begin{bmatrix}
\frac{dx_1}{dt} \\
\frac{dx_2}{dt} \\
\frac{dx_3}{dt} \\
\frac{dx_4}{dt} \\
\frac{dx_5}{dt} \\
\frac{dx_6}{dt}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
-k_{se} & -\frac{b_{se}}{M_{se}} & \frac{k_{se}}{M_{se}} & \frac{b_{se}}{M_{se}} & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
-k_{se} & b_{se} & -k_{se} & -k_s & 0 & \frac{k_s}{M_s} \\
0 & 0 & M_s & M_s & M_s & \frac{M_s}{M_s} \\
0 & 0 & \frac{k_s}{M_u} & b_s & \frac{k_s}{M_u} & \frac{k_t}{M_u} - \frac{b_s}{M_u}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\cdot u \quad (8)
\]

Using equations (7), we obtain according to [7] the output equation for displacement and velocity can be derived.
3. MODELLING THE PASSIVE SUSPENSION SYSTEM IN MATLAB SIMULINK

The block diagram presented in Figure 2 is designed for the Matlab Simulink simulation of a passive suspension system. We immediately notice the link that exists between the constitutive elements of the diagram and the presented mathematical model. The diagram allows visualizing the accelerations, speeds and movements for the driver and seat mass and unsprung mass at step-type and track irregularity stimulus signals. For this latter case there are multiple ways to achieve irregularities in the track surface, most often using a road that is simulated using randomize functions.

\[
y = \begin{bmatrix} Z_{sc} \\ Z_s \\ Z_u \\ \frac{dZ_{sc}}{dt} \\ \frac{dZ_s}{dt} \\ \frac{dZ_u}{dt} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ u \end{bmatrix}
\]

(9)

The solution that we suggest consists in reaching various “sensitive” areas that describe the problematic of track irregularities. Consequently, even if there might be an infinity of situations, usually a vehicle's wheels can encounter, while ascending or descending, the following categories of irregularities:

- step;
- steep ramp;
- smooth ramp;
- small irregularities with small share, at surface level;
- small irregularities with large share, at surface level;
- sinusoidal low frequency irregularities;
- sinusoidal high frequency irregularities;
- disastrous irregularities which cause the destruction of the drive system.

For the simulation we can observe, from Figure 3, that we first use a step signal.

![Fig. 3. Stimulus signals used in simulation.](image)

In Figure 3 we also observe the other categories of signals used for the simulation. The signals that simulate the unevenness in the drive track can be added, as mentioned above, in any configuration so that it will correspond to the need of its users.

4. RESULTS OBTAINED FOR THE SIMULATION OF THE PASSIVE SUSPENSION SYSTEM

In order to simulate in Matlab [8, 9] the behavior of the passive suspension system we chose, as initial parameters, the values mentioned in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$k_{se}$</th>
<th>$k_s$</th>
<th>$k_t$</th>
<th>$M_e$</th>
<th>$M_s$</th>
<th>$b_{se}$</th>
<th>$b_s$</th>
<th>$z_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>UM</td>
<td>N/m</td>
<td>N/m</td>
<td>N/m</td>
<td>kg</td>
<td>kg</td>
<td>kg</td>
<td>kg</td>
<td>kg</td>
</tr>
<tr>
<td>Value</td>
<td>8500</td>
<td>31000</td>
<td>127000</td>
<td>110</td>
<td>295</td>
<td>2850</td>
<td>1900</td>
<td>0.200</td>
</tr>
</tbody>
</table>

The simulation was achieved for the first two categories of signals mentioned in Figure 3, tracking the variations in movements, speeds and accelerations. In practice the $b_s$ coefficients may have different values within fairly wide limits, depending on the type of vehicle, engine displacement and weight. Table 2 [10], contains the average values of $b_s$ when the valves are closed, where $b_s = (b_{se} + b_{sd})/2$, relationship in which $b_{se}$ corresponds to the damper compression and $b_{sd}$ to its relaxation.

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>$b_s$</th>
<th>$b_d$</th>
<th>$b_s$</th>
<th>$b_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cars-engine displacement</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- very small</td>
<td>360</td>
<td>450</td>
<td>3000</td>
<td>3310</td>
</tr>
<tr>
<td>- small and medium</td>
<td>1030</td>
<td>900</td>
<td>3880</td>
<td>4100</td>
</tr>
<tr>
<td>- large</td>
<td>1380</td>
<td>920</td>
<td>4440</td>
<td>4470</td>
</tr>
<tr>
<td>Automotive truck – Weight $G_a$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G_a &lt; 9.0 \times 10^3$ N</td>
<td>1100</td>
<td>-</td>
<td>5870</td>
<td>-</td>
</tr>
<tr>
<td>$G_a &gt; 9.0 \times 10^3$ N</td>
<td>1160</td>
<td>1530</td>
<td>14300</td>
<td>11700</td>
</tr>
<tr>
<td>Bus with $G_a &gt; 9.0 \times 10^3$ N</td>
<td>800</td>
<td>900</td>
<td>13800</td>
<td>13400</td>
</tr>
</tbody>
</table>

Table 2. The average values of the damping coefficient $b_s$ [Ns/m].
Dixon and Mech [11] recommended that when using dampers, the asymmetry is taken into account as the relative amounts of bump and rebound damping, „which on passenger cars tends to be around 30/70, although not narrowly constrained, varying between 20/80 and 50/50. On motorcycles it seems to be even more asymmetric, perhaps from 20/80 to 5/95”.

Spring rate and damping are chosen according to comfort, road holding and handling specifications.

4.1. Simulating the passive suspension system in Matlab Simulink, step-type signal

In this scenario, the stimulus signal is step-type, the simulation outlining the consistency in the response of the passive suspension system to the input signal. In Figure 4 we can observe the evolution of the movement parameter for all three masses of the system.

![Graph of movement of masses in a step-type signal passive suspension system.](image)

**Fig. 4.** Movement of the masses in a step-type signal passive suspension system.

We immediately observe from Figure 4 that major differences appear regarding the movement of unsprung mass as opposed to sprung and seat mass. The curvature shape is totally different, presenting much smaller amplitudes in case of unsprung mass. The parameter values indicate movements of up to 0.158 m for seat and mass drive, 0.150 m for sprung mass and only 0.117 m for unsprung mass, whereas the minimum values are of 0.08 m both for sprung mass and for seat and drive mass, and 0.097 m for unsprung mass. We ascertain that the dampening of the oscillations occurs after 2.5 s from impact. We also notice that the signals for sprung mass and seat and drive mass are similar in shape and that their values differ very little.

![Graph of variation of movement speeds of the step-type signal masses.](image)

**Fig. 5.** Variation of the movement speeds of the step-type signal masses.

By analyzing the speed change when a step-type stimulus signal is applied in Figure 5 we observe that the velocity for unsprung mass is highly increased, given the fact that it is the first element of the suspension that takes over the shock. Moreover, we note that the speed with which the previously mentioned parameter increases is a lot higher. The maximum value of the velocity for unsprung mass is 3.214 m/s, for sprung mass 0.725 m/s and for seat and drive mass 0.642 m/s.
The acceleration of unsprung mass, as it results out of Figure 6 reaches a maximum of 32.78 m/s², a value much higher than the 2.7 m/s² for sprung mass or 1.8 m/s² for seat and drive mass. It can be observed that, in this case as well, the biggest oscillations appear for the unsprung mass, a result of the fact that this element is the most stressed.

4.2 Simulating the passive suspension system in Matlab Simulink, for an uneven track surface

In order to get as close as possible to the real scenario in which a passive suspension system functions on an uneven track surface, we used stimulus signal 2 from Figure 3 which, as it can be observed, allows analysis for the hypothesis in which the wheel passes over a smooth ramp followed by a steep ramp and a straight road leading to a ramp, a hole and another straight road.
Analyzing the details of the signals from Figure 7 we can ascertain that, when the wheel climbs on the first ramp, followed by straight road, the variations are much smaller as in the case when the step signal was applied, for all three masses of the suspension system.

On the other hand, in case of the second ramp, which is much steeper, the variations have bigger oscillations compared to the median area, reaching 0.035 m. When a wheel descends from a ramp, there are visible variations on the first ramp but, on the second, smoother ramp, the variations become noticeable when a step occurs as a result of track unevenness.

We ascertain that the passive suspension system usually follows the configuration of the track surface. From a value point of view we note that the variations are much smaller in case of the step-type signal stimulus.

The velocities for the uneven track surface, as it can be observed from Figure 8, show pin-type forms of signal every time the track surface suddenly changes its conformation. The highest values of the velocities are of 0.73 m/s when climbing the second ramp, respectively 0.88 m/s when descending the second ramp. Both values are for unsprung mass.
Fig. 9. Modifying the acceleration of passive suspension masses (A & B details) uneven track surface.

The accelerations show significant increases (Figure 9), especially for unsprung mass, the highest values being $0.64 \, \text{m/s}^2$ respectively $0.96 \, \text{m/s}^2$ for the same situations as in the case of velocity. For the passengers the accelerations do not last in time more than $0.18 \, \text{s}$ which indicates extremely short periods of time which become unnoticeable if are not repeated often.

5. CONCLUSIONS

In the present paper we presented a mathematical model of a passive suspension system, based on which we developed a simulation diagram using Matlab Simulink [9]. We aimed at analyzing the behavior of the seat and drive mass, sprung and unsprung mass, when the wheel faces various configurations of the surface track. As stimulus signals we used a step-type signal and a second, complex signal, corresponding to a surface track that has irregularities, simulating ramps of various amplitudes.

The analysis of the passive suspension system shows that, in order to ensure the best comfort it is necessary that:

- the vehicle wheels do not leave the track surface;
- the chassis resonance frequency does not pass $1 \, \text{Hz}$ and its peak is up to $10 \, \text{Hz}$;
- for the spring and damper suspension system it is recommended that the damper is soft with a deflection of its rod as high as possible in order to take over as much as possible road irregularities. However, in order to ensure that the wheel does not leave the track surface, the damper must be as hard as possible.

Based on the simulations we determined the parameters movement, velocity and acceleration for the three masses of the passive suspension system. The biggest movement was $0.158 \, \text{m}$ for seat and mass drive, the maximum value of the velocity for unsprung mass was of $3.214 \, \text{m/s}$, and the highest acceleration obtained by simulation was of $32.7 \, \text{m/s}^2$. All the maximal values correspond to a step-type signal.
Passive suspensions have inherent limitations as a consequence of the trade-off in the choice of the spring rate and damping characteristics [12, 13]. A linear system (spring–mass–damper) with a one-degree-of-freedom (1DOF) [12], modelled by a second-order linear differential equation, having high damping performs well in the vicinity of the resonant frequency and poorly far from it, whilst a low-damped system behaves conversely.

Matlab modelling allows changing the seat suspension spring stiffness, vehicle suspension spring stiffness, tire stiffness, damping ratio of the seat suspension and the damping ratio of the vehicle suspension. Simulation in Matlab using different values of these coefficients [14] allows studying the impact of the suspension on comfort and road holding and to compare them with the manufacturer’s handling specifications.

After performing simulations and choosing suspension coefficients it is necessary to ensure that the suspension unit ought to be able to reduce chassis acceleration as well as dynamic tyre force within the constraint of a set working space.

As shown, in the case of passive suspension a conflict clearly appears that can be resolved in two ways: the elastic and the damping characteristics are controlled closed-loop or by using an external power supply with feedback-controlled actuators.

In conclusion we can ascertain that establishing the parameters of a suspension system is a compromise between safety and comfort.

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