THERMO - MECHANICAL LOADING IN BEVELLED AREA BETWEEN TWO CYLINDRICAL SHELLS WITH DIFFERENT THICKNESSES.
THEORETICAL STUDY - CONNECTION LOADS

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Abstract. This paper addresses the problem of stress concentration in the transition area, between two cylindrical shells with different thicknesses. To reduce the intensity of the stresses and strains developed under the action of external mechanical and/or thermal loads, an original method is proposed, using method of short structural elements for shells, respectively bending moments theory and deformations continuity (displacements and rotations). In this sense, the transition between shell rings is considered linear variable in four constructive variants (with the same inner or outer surface, respectively the same median surface, or different median surfaces). Based on the evaluated intensity of the stress, it can be concluded which is the preferred variant the design stage, or deduction of the same sizes, in case of technological deviations (cutting errors, so as to result the same inner or outer surface, as in case of the same median surface). Article content refers in this configuration the setting mode related of connection loads that will be taken into account in tensions expressions.

Key words: cylindrical shell, short structural elements

1. INTRODUCTION

Avoidance of deterioration risk of any mechanical structures in general, and the structure under pressure and/or temperature, especially involves in particular, the quality of all pre-putting into operation actions and follow up normal operation. Therefore, an unavoidable conditioning is required by influences correlation imposed by conceiving (including research and design) - manufacture - transport - installation and, ultimately, maintenance [1, 2].

Lifetime of analyzed mechanical structure depends on a properly sized based on the use of some materials and certain performing technical solutions that enable judicious assessment of stresses and deformations values which are produced under the action of existing external loads and which are forecasted to be stable over time. In
the same sense, a given geometry can be capitalized at any time in order to determine the load-carrying capacity. Not only manufacturing technologies require a gradual transition (linear or fillet corner) from a thickness another of wall but also gained experience regarding attenuation of stress concentration in cylindrical and conical construction, smooth [3-15], respectively in construction with ribs [16, 17]. Reducing of stress intensity has a very favorable effect on the consumption of construction materials and exhausted energies over the life of the structure.

Fig. 1. Types of beveled zones between two successive cylindrical shells with different thicknesses: a - constant outer radius; b - constant inner radius; c - constant median radius (symmetrical beveled area); d - different medial surfaces (unsymmetrical beveled area).
This paper proposes the evaluation of loading status, in the crossing beveled zone, between two cylindrical shells with different thicknesses (Figure 1). Evaluation was performed based on the theory of bending moments, of revolution shells and of congruence deformations on the one hand, [3-5], and of method of short structural elements [9, 16, 17, 18], on the other hand. The paper sets out how the deduction of link loads, developed under the action of considered external loads, between shells and short structural elements (type cylindrical shell with length shorter than semiwave length [3-5], on the one hand, and between mentioned elements [9, 16-18], on the other hand. Some recognized normative [12, 19-24], make adequate specifications on the geometry of such beveled zones. Practice cannot meet always such recommendations, and as such it is required an atypical methodology for specific case.

2. HELPFUL SIZES

For achievement of the proposed aim, beveled zones of link are divided (Figure 2-5), between the two successive cylindrical shells, in any number of short cylindrical elements (blades). Considering $H$ - the zone height of a mentioned form, the $N$ - the number of the lamellar elements of equal height, $t$ (it is noted that the method also allows separation with different thickness which is accepted by researcher), it is deducted:

$$t = \frac{H}{N}$$ (1)
Other geometrical characteristics of lamellar elements are established as follows:

- **Shells with identical outer radius (Figure 1.a and Figure 2):**

\[
r_e = r_{i1} + \delta_1 = r_{i2} + \delta_2 \quad r_{m1} = r_{i1} + 0.5 \cdot \delta_1; \quad r_{m3} = r_{i3} + 0.5 \cdot \delta_3;
\]

\[
k_1 = \frac{\sqrt{3 \cdot (1 - \nu^2)}}{r_{m1} \cdot \delta_1}; \quad \mathfrak{R}_1 = \frac{E \cdot \delta_1}{12 \left(1 - \nu^2\right)};
\]

\[
\tan \alpha = (\delta_3 - \delta_1) / H = \theta; \quad r_{i21} = r_{i1} - \frac{1}{2} \cdot t \cdot \theta; \quad \delta_{21} = r_e - r_{i21}; \quad r_{m21} = 0.5 \left(r_e + r_{i21}\right);
\]

\[
k_{21} = \frac{\sqrt{3 \cdot (1 - \nu^2)}}{r_{m21} \cdot \delta_{21}}; \quad \mathfrak{R}_{21} = \frac{E \cdot \delta_{21}}{12 \left(1 - \nu^2\right)}; \quad r_{j2j} = r_{i1} - \left(j - \frac{1}{2}\right) \cdot t \cdot \theta;
\]

\[
\delta_{2j} = r_e - r_{i2j}; \quad r_{m2j} = 0.5 \left(r_e + r_{i2j}\right); \quad k_{2j} = \frac{\sqrt{3 \cdot (1 - \nu^2)}}{r_{m2j} \cdot \delta_{2j}}; \quad \mathfrak{R}_{2j} = \frac{E \cdot \delta_{2j}}{12 \left(1 - \nu^2\right)};
\]

\[j \in \{2, ..., N - 1\}; \quad r_{i2N} = r_{i1} - \left(N - \frac{1}{2}\right) \cdot t \cdot \theta;
\]

\[
\delta_{2N} = r_{e2N} - r_e; \quad r_{m2N} = 0.5 \left(r_e + r_{e2N}\right);
\]

\[
k_{2N} = \frac{\sqrt{3 \cdot (1 - \nu^2)}}{r_{m2N} \cdot \delta_{2N}}; \quad \mathfrak{R}_{2N} = \frac{E \cdot \delta_{2N}}{12 \left(1 - \nu^2\right)}; \quad k_3 = \frac{\sqrt{3 \cdot (1 - \nu^2)}}{r_{m3} \cdot \delta_3}; \quad \mathfrak{R}_3 = \frac{E \cdot \delta_3}{12 \left(1 - \nu^2\right)} \quad (2)
\]

**Note 1:** As a calculation base, the outer radius of cylindrical shells or inner radius of cylindrical shell 1 (as above), respectively the inner radius of cylindrical shell 3 can be taken into account:

- **Shell rings with inner radius identical (Figure 1.b and Figure 3):**

\[
r_{r1} = r_{i1} + \delta_1; \quad r_{m1} = r_{i1} + 0.5 \cdot \delta_1; \quad r_{r3} = r_{i3} + \delta_3;
\]

\[
r_{m3} = r_{i3} + 0.5 \cdot \delta_3; \quad \tan \alpha_2 = (\delta_3 - \delta_1) / H = \theta_2; \quad r_{e21} = r_{e1} + \frac{1}{2} \cdot t \cdot \theta; \quad \delta_{21} = r_{e21} - r_{i1};
\]

\[
r_{m21} = 0.5 \left(r_{e21} + r_{i1}\right); \quad k_{21} = \frac{\sqrt{3 \cdot (1 - \nu^2)}}{r_{m21} \cdot \delta_{21}}; \quad \mathfrak{R}_{21} = \frac{E \cdot \delta_{21}}{12 \cdot (1 - \nu^2)}; \quad r_{e2j} = r_{e1} + \left(j - \frac{1}{2}\right) \cdot t \cdot \theta;
\]

\[
\delta_{2j} = r_{e2j} - r_{i1}; \quad r_{m2j} = 0.5 \left(r_{e2j} + r_{i1}\right); \quad k_{2j} = \frac{\sqrt{3 \cdot (1 - \nu^2)}}{r_{m2j} \cdot \delta_{2j}}; \quad \mathfrak{R}_{2j} = \frac{E \cdot \delta_{2j}}{12 \left(1 - \nu^2\right)};
\]

\[j \in \{2, ..., N - 1\}; \quad r_{e2N} = r_{e1} + \left(N - \frac{1}{2}\right) \cdot t \cdot \theta;
\]
\[
\delta_{2N} = r_{e2N} - r_i; \quad r_{m2N} = 0.5 \left( r_i + r_{e2N} \right); \quad k_{2N} = \frac{\sqrt{3 \cdot (1 - \nu^2)}}{\sqrt{r_{m2N} \cdot \delta_{2N}}}; \quad \gamma_{2N} = \frac{E \cdot \delta_{2N}^3}{12 \left( 1 - \nu^2 \right)} \tag{3}
\]

- **Shell rings with equal radius to of median surface and intermediate zone with identical bevels** (Figure 1.c and Figure 4):

\[
r_{m1} = r_{m3} = r_m = r_{e1} + 0.5 \cdot \delta_1 = r_{i3} + 0.5 \cdot \delta_3; \quad \tan \alpha_1 = 0.5 \left( \delta_3 - \delta_1 \right)/H = \theta_1; \quad \\
\frac{r_{i21}}{r_{e21}} = \frac{1}{2} \cdot \vec{t} \cdot \vec{\theta}_1; \quad \frac{r_{e21}}{r_{i21}} = \frac{1}{2} \cdot \vec{t} \cdot \vec{\theta}_1; \quad \delta_{21} = r_{e21} - r_{i21}; \quad r_{m21} = 0.5 \left( r_{e21} + r_{i21} \right); \\
k_{21} = \frac{\sqrt{3 \cdot (1 - \nu^2)}}{\sqrt{r_{m21} \cdot \delta_{21}}}; \quad \gamma_{21} = \frac{E \cdot \delta_{21}^3}{12 \left( 1 - \nu^2 \right)}; \quad r_{i2j} = r_{e1} - \left( j - \frac{1}{2} \right) \cdot \vec{t} \cdot \vec{\theta}_1; \quad j \in \{ 2, ..., N - 1 \}; \\
\delta_{2j} = r_{e2j} - r_{i2j}; \quad k_{2j} = \frac{\sqrt{3 \cdot (1 - \nu^2)}}{\sqrt{r_{m2j} \cdot \delta_{2j}}}; \quad \gamma_{2j} = \frac{E \cdot \delta_{2j}^3}{12 \left( 1 - \nu^2 \right)}; \quad j \in \{ 2, ..., N - 1 \}; \\
r_{e2N} = r_{e1} + \left( N - \frac{1}{2} \right) \cdot \vec{t} \cdot \vec{\theta}_1; \quad r_{i2N} = r_{i1} - \left( N - \frac{1}{2} \right) \cdot \vec{t} \cdot \vec{\theta}_1; \quad \delta_{2N} = r_{e2N} - r_{i2N}; \\
k_{2N} = \frac{\sqrt{3 \cdot (1 - \nu^2)}}{\sqrt{r_{m2N} \cdot \delta_{2N}}}; \quad \gamma_{2N} = \frac{E \cdot \delta_{2N}^3}{12 \left( 1 - \nu^2 \right)} \tag{4}
\]

- **Shell rings with unequal median surfaces and intermediate zone with different bevels** (Figure 1.d and Figure 5):

\[
r_{e1} = r_{e1} + \delta_1; \quad r_{m1} = r_{m1} + 0.5 \cdot \delta_1; \quad r_{e3} = r_{e3} + \delta_3; \quad r_{m3} = r_{m3} + 0.5 \cdot \delta_3; \quad e_1 = r_{e1} - r_{i1}; \\
e_2 = r_{e3} - r_{e1}; \quad \tan \alpha_2 = e_1 / H = \theta_2; \quad \tan \alpha_3 = e_2 / H = \theta_3; \\
r_{i21} = r_{e1} - \frac{1}{2} \cdot \vec{t} \cdot \vec{\theta}_2; \quad r_{e21} = r_{e1} + \frac{1}{2} \cdot \vec{t} \cdot \vec{\theta}_3; \quad \delta_{21} = r_{e21} - r_{i21}; \quad r_{m21} = 0.5 \left( r_{e21} + r_{i21} \right); \\
k_{21} = \frac{\sqrt{3 \cdot (1 - \nu^2)}}{\sqrt{r_{m21} \cdot \delta_{21}}}; \quad \gamma_{21} = \frac{E \cdot \delta_{21}^3}{12 \left( 1 - \nu^2 \right)}; \quad r_{i2j} = r_{e1} - \left( j - \frac{1}{2} \right) \cdot \vec{t} \cdot \vec{\theta}_2; \\
\delta_{2j} = r_{e2j} - r_{i2j}; \quad k_{2j} = \frac{\sqrt{3 \cdot (1 - \nu^2)}}{\sqrt{r_{m2j} \cdot \delta_{2j}}}; \quad \gamma_{2j} = \frac{E \cdot \delta_{2j}^3}{12 \left( 1 - \nu^2 \right)}; \quad j \in \{ 2, ..., N - 1 \};
\[ r_{e2N} = r_{e1} + \left( N - \frac{1}{2} \right) \cdot \theta \cdot r_1 \cdot \delta_{2N} = r_{i1} - \left( N - \frac{1}{2} \right) \cdot \theta \cdot r_1 \cdot \delta_{2N} = r_{e2N} = r_{i2N} \]

\[ k_{2N} = \sqrt[3]{\frac{\left( 1 - \nu^2 \right)}{r_{m2N} \cdot \delta_{2N}}} \quad \Re_{2N} = \frac{E \cdot \delta_{2N}^3}{12 \left( 1 - \nu^2 \right)} \]  

(5)

**Note 2:** For the performed calculations in the present work, the following conditions must be satisfied

\[ \delta_{1} / r_{i1} < 0,2 ; \quad \delta_{3} / r_{i3} < 0,2 \]  

(6)

which includes all short structural elements in the category of revolution shells [2-5].

### 3. EXTERNAL LOADS

It is considered, in what follows, that pressure dependent on current quota \( x \), measured along the generators, acts on cylindrical shells and the beveled transition zone, regardless of the existing shape (Figure 1):

\[ p(x) = p_i(x) - p_e(x) \]  

(7)

Pressure includes the hydrostatic pressure developed inside and/or outside of structure. In this way, it is possible to calculate the pressure at the base of cylindrical zone \( I \) (Figure 1) by the expression:

\[ p(x_1 = 0) = p_{0i} - p_{0e} + g \left[ \rho_{r_1} \cdot H_{i1}(x_1 = 0) - \rho_{r_e} \cdot H_{i1}(x_1 = 0) \right] \]  

(8)

At the level of short structural element with sequence number \( j (j \in \{1, 2, \ldots, N\}) \), design pressure \( p(j) \) is determined by the expression

\[ p(j) = p(x_1 = 0) + \frac{2 \cdot j - 1}{2} \left( \rho_{i1} - \rho_{i e} \right) \cdot g \cdot t \]  

(9)

A similar method can be accepted for the internal temperature of the working environment \( T_i(x) \) and outside temperature \( T_e(x) \) so it is deduced wall temperature average (thin shell):

\[ T(x) = 0.5 \left[ T_i(x) + T_e(x) \right] \]  

(10)

**Note 3:** Pressure and temperature in the cylindrical shell rings (Figure 1) are accepted with the same value for the entire length/height of the beveled zone that links the two cylindrical shell rings

Because within presented analysis, the structure is in vertical position, so that axially symmetric loads are accepted and axial force which is positioned at the base at cylindrical shell ring, \( I \) is determined and it is written in the form:

\[ F_{1i} = \frac{1}{2} \left[ p(x_1 = 0) \cdot r_{m1} + \frac{1}{\pi \cdot r_{m1}} \sum F_{a1} \right] \]  

(11)

Keeping axial force constant, unitary axial forces created for each short structural element, can be calculated with the expression:
Fig. 6. Scheme of separation elements 1 and 2.

\[ \vec{P}_{2j} = \left( \frac{r_{m1}}{r_{m2j}} \right) \vec{P}_1 \]  

(12)

while for shell ring 3:

\[ \vec{P}_3 = \left( \frac{r_{m1}}{r_{m3}} \right) \vec{P}_1 \]  

(13)

Note 4: In the above analysis it was considered that the structure is propped under the beveled zone. When the propping is above the mentioned zone, unitary axial forces, previously mentioned, must be adapted adequately.

4. CONTINUITY / COMPATIBILITY EQUATIONS FOR DEFORMATIONS

All the assumptions for calculating specific revolution shells are accepted [3-5]. For finer calculations, variations of axial load can be established, in each short structural element, as well as temperature variation. In this regard, appropriate relationships are adapted to the considered estimation. Considering the thickness \( t \), relatively small, of short structural elements, we neglect the influence of radial pressure on their rotation, into the equalities accepted for deformations.

Note 5: Sign convention for deformations (radial displacements and rotations), at theirs equation of continuity is: radial displacements are considered negative when they produce an increase of the radius in the plan; rotations are positive, with clockwise, both the bottom of the above as well as below of the top of the element, for the considered connection (for example: \( 1 - 2 \), \( 2 - 3 \), \( 2_{N-1} - 2_2 \), \( 2_2 - 3 \)).

Based on the deformation compatibility equations of merge features set, you can write the following equalities:

- Merged elements 1 – 2 (Figure 6):

\[ a(1,1) \cdot M_{01} + a(1,2) \cdot Q_{01} + a(1,3) \cdot M_{02} + a(1,4) \cdot Q_{02} = b(1) \]  

(14)

\[ a(2,1) \cdot M_{01} + a(2,2) \cdot Q_{01} + a(2,3) \cdot M_{02} + a(2,4) \cdot Q_{02} = b(2) \]  

(15)
- **Merged elements** $2_{j-1} - 2_j$ (Fig. 7) - $j \in \{2, \cdots, N\}$:

$$a(2 \cdot j - 1, j - 1) \cdot M_{0(j-1)} + a(2 \cdot j - 1, j) \cdot Q_{0(j-1)} + a(2 \cdot j - 1, j + 1) \cdot M_{0j} + a(2 \cdot j - 1, j + 2) \cdot Q_{0j} + a(2 \cdot j - 1, j + 3) \cdot M_{0(j+1)} + a(2 \cdot j - 1, j + 4) \cdot Q_{0(j+1)} = b(2 \cdot j - 1)$$ (16)

$$a(2 \cdot j, j - 1) \cdot M_{0(j-1)} + a(2 \cdot j, j) \cdot Q_{0(j-1)} + a(2 \cdot j, j + 1) \cdot M_{0j} + a(2 \cdot j, j + 2) \cdot Q_{0j} + a(2 \cdot j, j + 3) \cdot M_{0(j+1)} + a(2 \cdot j, j + 4) \cdot Q_{0(j+1)} = b(2 \cdot j)$$ (17)

![Fig. 7. Short structural separation scheme](image)

Fig. 7. Short structural separation scheme $2_j$ and $2_{j+1}$

- **Merged elements** $2_{2N} - 3$ (Figure 8):

$$a(2 \cdot N + 1, N + 1) \cdot M_{0N} + a(2 \cdot N + 1, N + 2) \cdot Q_{0N} + a(2 \cdot N + 1, N + 3) \cdot M_{0(N+1)} + a(2 \cdot N + 1, N + 4) \cdot Q_{0(N+1)} = b(2 \cdot N + 1)$$ (18)

$$a(2 \cdot N + 2, N + 1) \cdot M_{0N} + a(2 \cdot N + 2, N + 2) \cdot Q_{0N} + a(2 \cdot N + 2, N + 3) \cdot M_{0(N+1)} + a(2 \cdot N + 2, N + 4) \cdot Q_{0(N+1)} = b(2 \cdot N + 2)$$ (19)

Combining the equations of continuity / compatibility of deformations between structural elements, mentioned in the above, we can write, in summary form:

$$\begin{bmatrix} a_{mn} \end{bmatrix} \cdot \{\square\} = \{b(m)\}$$ (20)

from which it follows:

$$\{\square\} = \begin{bmatrix} a_{mn} \end{bmatrix}^{-1} \cdot \{b(m)\}$$ (21)
Fig. 8. Separation scheme elements $2_N$ and $3$

**Note 6**: Into the equalities (20) and (21) the following meanings are used:

- $\left[a_{mn}\right]$ – The matrix of influence factors $(m \in \{1, \cdots, 2 \cdot N + 2\}; n \in \{1, \cdots, 2 \cdot N + 2\})$, respectively:

  \[
  a(1, 1) = -\frac{1}{2 \cdot k_2^2 \cdot r_{11}} + \frac{1}{2 \cdot k_2^2 \cdot r_{21}} \cdot \frac{r_{m1}}{r_{m1}}, f_{1m}(1)
  \]

  \[
  a(1, 2) = -\frac{1}{2 \cdot k_2^2 \cdot r_{11}} + \frac{1}{2 \cdot k_2^2 \cdot r_{21}} \cdot \frac{r_{m1}}{r_{m1}}, f_{1q}(1)
  \]

  \[
  a(1, 3) = \frac{1}{2 \cdot k_2^2 \cdot r_{21}} \cdot f_{md}(1); \quad a(1, 4) = \frac{1}{2 \cdot k_2^2 \cdot r_{21}} \cdot f_{qd}(1)
  \]

  \[
  a(2, 1) = \frac{1}{2 \cdot k_2^2 \cdot r_{11}} - \frac{1}{k_2^2 \cdot r_{21}} \cdot \frac{r_{m1}}{r_{m1}}, f_{2m}(1)
  \]

  \[
  a(2, 2) = \frac{1}{2 \cdot k_2^2 \cdot r_{21}} - \frac{1}{2 \cdot k_2^2 \cdot r_{21}} \cdot \frac{r_{m1}}{r_{m1}}, f_{2q}(1);
  \]

  \[
  a(2, 3) = \frac{1}{2 \cdot k_2^2 \cdot r_{21}} \cdot f_{mr}(1); \quad a(2, 4) = \frac{1}{2 \cdot k_2^2 \cdot r_{21}} \cdot f_{qr}(1)
  \]

  \[
  j \in \{2, \cdots, N\}
  \]

  \[
  a(2 \cdot j - 1, j - 1) = -\frac{1}{2 \cdot k_2^2 \cdot \gamma_{2(j-1)}}, \frac{r_{m1}}{r_{m1}}, f_{md}(j - 1)
  \]
\begin{align}
a(2 \cdot j - 1, j) &= (-1)^{j-1} \cdot \frac{1}{2 \cdot k_{2(j-1)}^3 \cdot R_{2(j-1)}} \cdot \frac{r_{m1}}{r_{m2(j-1)}} \cdot f_{q_d}(j - 1) \\
a(2 \cdot j - 1, j + 1) &= -\frac{1}{2 \cdot k_{2(j-1)}^2 \cdot R_{2(j-1)}} \cdot f_{1m}(j - 1) + \frac{1}{2 \cdot k_{2j}^3 \cdot R_{2j}} \cdot \frac{r_{m1}}{r_{m2j}} \cdot f_{1m}(j) \\
a(2 \cdot j - 1, j + 2) &= (-1)^{j-1} \left[ \frac{1}{2 \cdot k_{2(j-1)}^3 \cdot R_{2(j-1)}} \cdot f_{1q}(j - 1) + \frac{1}{2 \cdot k_{2j}^3 \cdot R_{2j}} \cdot \frac{r_{m1}}{r_{m2j}} \cdot f_{1q}(j) \right] \\
a(2 \cdot j - 1, j + 3) &= \frac{1}{2 \cdot k_{2j}^2 \cdot R_{2j}} \cdot f_{m_d}(j) \\
a(2 \cdot j - 1, j + 4) &= (-1)^{j-1} \frac{1}{2 \cdot k_{2j}^3 \cdot R_{2j}} \cdot f_{q_d}(j) \\
a(2 \cdot j, j - 1) &= -\frac{1}{2 \cdot k_{2(j-1)}^3 \cdot R_{2(j-1)}} \cdot \frac{r_{m1}}{r_{m2(j-1)}} \cdot f_{m_r}(j - 1) \\
a(2 \cdot j, j) &= -(-1)^{j-1} \cdot \frac{1}{2 \cdot k_{2j}^3 \cdot R_{2j}} \cdot \frac{r_{m1}}{r_{m2j}} \cdot f_{q_r}(j - 1) \\
a(2 \cdot j, j + 1) &= -\frac{1}{k_{2(j-1)} \cdot R_{2(j-1)}} \cdot f_{2m}(j - 1) - \frac{1}{k_{2j} \cdot R_{2j}} \cdot \frac{r_{m1}}{r_{m2j}} \cdot f_{2m}(j) \\
a(2 \cdot j, j + 2) &= (-1)^{j-1} \left[ \frac{1}{2 \cdot k_{2(j-1)}^3 \cdot R_{2(j-1)}} \cdot f_{23q}(j - 1) - \frac{1}{2 \cdot k_{2j}^3 \cdot R_{2j}} \cdot \frac{r_{m1}}{r_{m2j}} \cdot f_{23q}(j) \right] \\
a(2 \cdot j, j + 3) &= \frac{1}{2 \cdot k_{2j}^2 \cdot R_{2j}} \cdot f_{m_r}(j) \\
a(2 \cdot j, j + 4) &= (-1)^{j-1} \frac{1}{2 \cdot k_{2j}^3 \cdot R_{2j}} \cdot f_{q_r}(j) \\
a(2 \cdot N + 1, N) &= -\frac{1}{2 \cdot k_{2N}^3 \cdot R_{2N}} \cdot \frac{r_{m1}}{r_{m2N}} \cdot f_{m_d}(N) \\
a(2 \cdot N + 1, N + 1) &= -(-1)^{N-1} \frac{1}{2 \cdot k_{2N}^3 \cdot R_{2N}} \cdot \frac{r_{m1}}{r_{m2N}} \cdot f_{q_d}(N) \\
a(2 \cdot N + 1, N + 2) &= -\frac{1}{2 \cdot k_{2N}^2 \cdot R_{2N}} \cdot f_{1m}(N) + \frac{1}{2 \cdot k_{3j}^3 \cdot R_{3j}} \cdot \frac{r_{m1}}{r_{m3}}
\end{align}
\[ a(2 \cdot N + 1, N + 3) = - (1)^{N-1} \left[ \frac{1}{2 \cdot k^4_{0 \cdot N}} \cdot f_{1q} (N) - \frac{1}{2 \cdot k^4_{0 \cdot 3}} \cdot \frac{r_{m1}}{r_{m3}} \right] \]  
(43)

\[ a(2 \cdot N + 2, N) = - \frac{1}{2 \cdot k^4_{0 \cdot N}} \cdot \frac{r_{m1}}{r_{m2}} \cdot f_{1q} (N) \]  
(44)

\[ a(2 \cdot N + 2, N + 1) = - (1)^{N-1} \left[ \frac{1}{2 \cdot k^4_{0 \cdot N}} \cdot \frac{r_{m1}}{r_{m2}} \cdot f_{1q} (N) \right] \]  
(45)

\[ a(2 \cdot N + 2, N + 2) = - \frac{1}{k^4_{0 \cdot N}} \cdot \frac{r_{m1}}{r_{m3}} \cdot f_{2m} (N) - \frac{1}{k^4_{0 \cdot 3}} \cdot \frac{r_{m1}}{r_{m3}} \]  
(46)

\[ a(2 \cdot N + 2, N + 3) = - (1)^{N-1} \left[ \frac{1}{2 \cdot k^4_{0 \cdot N}} \cdot f_{23q} (N) + \frac{1}{2 \cdot k^4_{0 \cdot 3}} \cdot \frac{r_{m1}}{r_{m3}} \right] \]  
(47)

\[ \{ \square \}^T = \{ M_{0 \cdot 1}, Q_{0 \cdot 1}, M_{0 \cdot 2}, Q_{0 \cdot 2}, \ldots, M_{0 \cdot j}, Q_{0 \cdot j}, \ldots, M_{0 \cdot (N + 1)}, Q_{0 \cdot (N + 1)} \} \] - transposed vector of unknowns current problem - connected loads - radial and unitary bending moment and cutting efforts;

\[ \{ b(m) \}^T = \{ b(1), b(2), \ldots, b(j-1), b(j), \ldots, b(2 \cdot N + 1), b(2 \cdot N + 2) \} \] - transposed vector of free terms of the algebraic system (20) or (21) - with the significance of radial displacements and rotations caused by external loads acting on the structure:

\[ b(1) = \frac{p(x_j = 0)}{4} \left[ 1 - \frac{1}{k^4_{1 \cdot R_1}} \right] - \frac{\nu \cdot \bar{P}_1}{4} \left[ 1 - \frac{1}{k^4_{1 \cdot R_1}} \cdot \frac{r_{m1}}{r_{m21}} \right] \]  
(48)

\[ b(2) = \frac{d}{d x} \left[ p(x) \right]_{x_j = 0} - \alpha \cdot \frac{d}{d x} \left[ T(x) \right]_{x_j = 0} - \frac{\nu}{4 \cdot k^4_{1 \cdot R_1}} \cdot \frac{r_{m1}}{r_{m21}} \left[ \frac{d}{d x} \left[ \bar{P}_1(x) \right] \right]_{x_j = 0} \]  
(49)

**Note 7:** When the pressure, temperature or axial load - or one of them - is not considered dependent on derivative / derivatives from the equality (23) will be, corresponding, canceled.

\[ j \in \{ 2, \ldots, N \} \]

\[ b(2 \cdot j - 1) = \frac{p(x_j = 0)}{4} \left[ 1 - \frac{1}{k^4_{2 \cdot (j-1) \cdot R_{2 \cdot (j-1)}}} \right] - \frac{\nu \cdot \bar{P}_1 \cdot r_{m1}}{4} \left[ 1 - \frac{1}{k^4_{2 \cdot j \cdot R_{2 \cdot j}}} \cdot \frac{r_{m2 \cdot (j-1)}}{r_{m2 \cdot 2 \cdot j}} \right] \]  
(50)

\[ b(2 \cdot j) = 0 \]  
(51)
\[ b(2N + 1) = \frac{p(x_1 = 0)}{4} \left( \frac{1}{k_2^{\text{e}} R_2^{\text{e}}} - \frac{1}{k_3^{\text{e}} R_3^{\text{e}}} \right) - \frac{v \cdot P_x}{4} \left( \frac{1}{k_2^{\text{e}} R_2^{\text{e}} - r_m^{\text{e}} R_2^{\text{e}} - r_m^{\text{e}} R_3^{\text{e}}} \right) + \alpha \sigma (r_m^{\text{e}} - r_m^{\varepsilon}) \Delta T(x_1 = 0) \] 

\[ b(2N + 2) = -\frac{1}{4 \cdot k_3^{\text{e}} R_3^{\text{e}}} \frac{d p(x_3)}{d x_3} - \alpha \sigma (r_m^{\text{e}} - r_m^{\varepsilon}) \Delta T(x_3) \frac{d T(x_3)}{d x_3} - \frac{v}{4 \cdot k_3^{\text{e}} R_3^{\text{e}} r_m^{\varepsilon}} \frac{d P_x}{d x_3} \] 

(52) 

(53) 

**Note 8:** Observations in note 7 remain valid for derived derivatives from equalities (52) and (53), relative to the current variable \( x_3 \). 

However, in the previous equalities, the following notations have been used:

\[ f_{1m}(j) = \left[ \frac{\sin^2(k_j t) + \sin^2(k_j t)}{N(j)} \right] \] 

\[ f_{2m}(j) = -\left[ \frac{\sin(k_j t) \cdot \cosh(k_j t) + \sin(k_j t) \cdot \cos(k_j t)}{N(j)} \right] \] 

\[ f_{2m}(j) = f_{1m}(j) \cdot \cos(k_j t) \cdot \cosh(k_j t) + + f_{2m}(j) \left[ \frac{\sin(k_j t) \cdot \cosh(k_j t) + \sin(k_j t) \cdot \cosh(k_j t)}{N(j)} \right] \] 

\[ f_{1m}(j) = f_{1m}(j) \cdot \left[ \cos(k_j t) \cdot \sin(k_j t) - \sin(k_j t) \cdot \cosh(k_j t) \right] \] 

\[ f_{2m}(j) = f_{1m}(j) \cdot \cos(k_j t) \cdot \sin(k_j t) + \sin(k_j t) \cdot \cosh(k_j t) + \cos(k_j t) \cdot \sin(k_j t) \] 

\[ f_{1q}(j) = \left[ \frac{\sin(k_j t) \cdot \cosh(k_j t) - \sin(k_j t) \cdot \sinh(k_j t)}{N(j)} \right] \] 

\[ f_{2m}(j) = \left[ \frac{\sin^2(k_j t)}{N(j)} \right] \] 

\[ f_{2m}(j) = \frac{f_{1q}(j) \cdot \cos(k_j t) \cdot \cosh(k_j t) + \frac{f_{2q}(j) \cdot \cos(k_j t) \cdot \sinh(k_j t)}{N(j)} + + f_{3q}(j) \cdot \sin(k_j t) \cdot \cosh(k_j t)}{N(j)} \] 

\[ f_{2q}(j) = \frac{f_{1q}(j) \cdot \sin(k_j t) \cdot \sinh(k_j t)}{N(j)} \] 

\[ f_{2q}(j) = \frac{f_{1q}(j) \cdot \sin(k_j t) \cdot \sinh(k_j t) - \sin(k_j t) \cdot \sinh(k_j t)}{N(j)} + + f_{2q}(j) \cdot \sin(k_j t) \cdot \sinh(k_j t) + + f_{3q}(j) \cdot \sin(k_j t) \cdot \sinh(k_j t) \] 

(54) 

(55) 

(56) 

(57) 

(58) 

(59) 

(60) 

(61) 

(62)
Other used notations:

\( g \) – acceleration of gravity; \( j \) – current number of lamellar element (Figures 2 – 5); \( k \) – considered mitigation factor of the short structural element; \( p_i (x), p_j (x) \) – the inner and outer pressure, dependent to the current quota \( x \); \( p_{oi}, p_{oe} \) – overpressure of gaseous media above the liquid on the inside and on the outside of the structure; \( r_{m1}, r_{m2}, r_{m3} \) – the radius of the median surfaces of the cylindrical shells 1 and 2 (Figure 1), respectively the average radius of a short structural element (Figures 2 – 5); \( t \) – thickness of a short structural element \((t = H / N)\); \( x_1, x_3 \) – current quotas discussed at the assessment of tensions developed along cylindrical walls (noted by 1, respectively by 3 – Figure 1); \( E \) – longitudinal modulus of elasticity of the material (it is considered that both cylindrical shells and the passage, have the same elastic characteristics of the materials); \( F_{ax} \) – defined axial forces by the weight of the construction material of the structure above the quota \( x_1 \), weight thermal insulation or other axial loads developed during the function time (for example: produced in connections, thermal effects); \( H \) – height of beveled area; \( H_1, H_e \) – height of the liquid inside the structure, respectively the liquid height on the outside of the structure, both values can be taken into account in calculating the corresponding hydrostatic pressures; \( H_1(x_i = 0), H_e(x_i = 0) \) – the heights of the columns of fluid inside the structure and the outside, to the quota \( x_i = 0 \) (fig. 1); \( N \) – number of short structural elements (beams); \( T_i(x), T_e(x) \) – the inner and external temperature-dependent current quota \( x \); \( T_o \) – ambient temperature (reference temperature); \( \alpha \) – rake angle of the beveled surfaces in relation to generators cylindrical surfaces of shells; \( \alpha_r \) – heat deformation factor; \( \delta_1, \delta_3 \) – thickness of cylindrical shells (Figure 1); \( \theta \) – tangent of the angle of the beveled surfaces; \( \nu \) – transverse coefficient of contraction (Poisson) for construction materials; \( \rho_{i1}, \rho_{i3} \) – density of the fluid inside the structure, respectively of the liquid from the outside of the structure; \( \Delta T(x) \) – thermal gradient \([\Delta T(x) = T(x) - T_0]\), rated for both current quota \( x_1 \), respectively \( x_3 \); \( \Re \) – cylindrical bending stiffness of the short structural element briefly considered \((\Re_1, \Re_{2j} - j \in \{1, N\} -, \Re_3)\);

5. CONCLUSIONS

This paper aims to present an original methodology based on the theory of unitary bending moments, characteristic for shells of revolution, respectively shorter structural theory. In this sense it pursues, to the present case, the determination of related loads (unitary radial bending moments, cutting efforts) in the separation plans of the structure elements, with transition areas from one thickness to another, with linear variation (the four cases analyzed). Their values can be used in subsequent works, at the evaluation of the average radial and annular stress, respectively of the maximal equivalent stress. Based on its value it can be concluded if the structure is able to operate or it’s necessary to go to specific adaptations: changing the construction material or the geometry used in the study.

An interesting observation is that the above results can be adjusted accordingly, when the switch between two different thicknesses of the shells are made through connections with identical or different geometry.

REFERENCES

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