PROFILING OF THE HOB TOOL FOR WORM SHAFTS DEFORMATION

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Abstract: The paper presents an algorithm for profiling the hob tool designed to generate by plastic deformation the worm shafts from the composition of worm-wheel gear type, used in the seat adjustment mechanisms of some Audi and Mercedes cars. The active surface of the hob tool is a cylindrical helical surface of constant pitch. Two such tandem tools are used for deformation, the semi-finished product being positioned between them. During generation, the tools rotate around their own axes, which, combined with their helical surface, causes a helical movement of the blank. The active surfaces of the tools are mutually winding on the helical flanks of the generated worm. In the paper, two applications were developed for the generation of worms whose dimensions were determined by 3D scanning.

Keywords: „virtual pole" method, hob tool, helical surfaces

1. INTRODUCTION

Basically, the generating tool of a semi-finished product bounded by a helical surface, a tool that processes by the method of point contact (Olivier's theorem II) is a supposedly cylindrical helical surface, of constant pitch, mutually enwrapping with helical surface [1, 2].

A principial solution by the "virtual pole" method [3] is analyzed in two successive steps:
1. Determination of the intermediate surface in contact with the blank (generating rack-gear tool), in the form of a linear contact;
2. Being known the generating rack-gear tool, its linear contact with the primary peripheral surface of the future hob tool is determined [4-6].

The two contact lines, the blank- rack tool and the hob primary peripheral surface-rack tool may coincide or intersect at a point called the characteristic point [7].

This analysis is proposed for a separate tool, the deformation tool which generate a cylindrical helical surface of constant pitch. The two helical surfaces have parallel axes.

Reference systems (Figure 1) are defined:
- XYZ is a mobile system, associated with the surface of the semi-finished product and the C_1 centrode;
- xyz - fixed system, associated with the axis of the blank;
- $\xi\eta\varsigma$ mobile system, associated with the rack and the centroid C_2;
- $x_1y_1z_1$ – fixed reference system, of the hob tool;

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- $X_1Y_1Z_1$ - mobile system, associated with the hob tool and with the C_3 centrode.

The following motion parameters are defined: φ1 is the angle of rotation of the centroid C_1, λ - the parameter of the translational motion of the running plane of the rack and of the centroid C_2, φ_2 - the angle of rotation of the centrode C_3, joined to the surface of the screw tool.

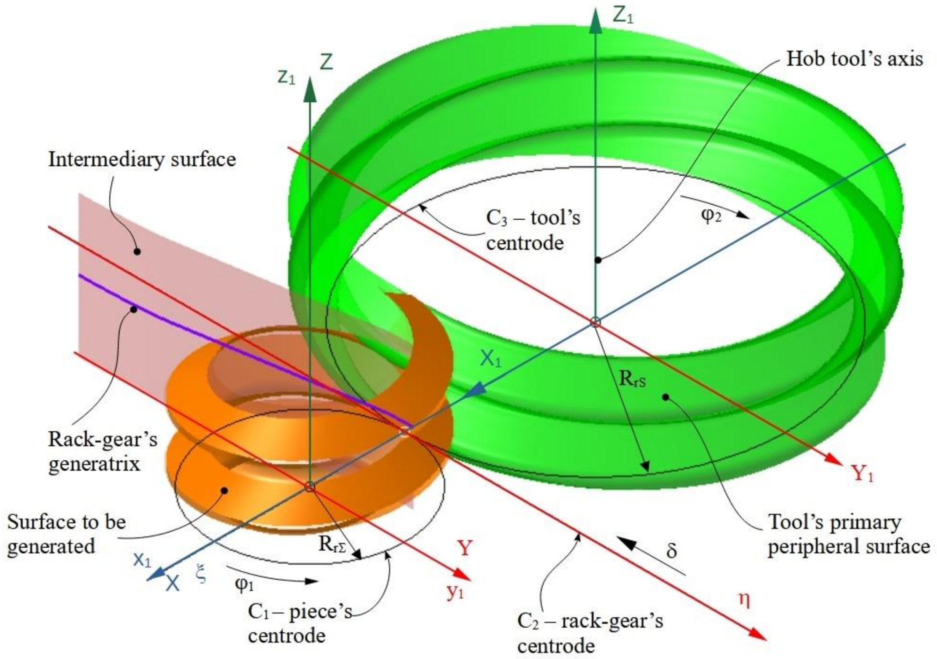


Fig. 1. Rolling centrodes; reference systems.

2. DETERMINATION OF THE INTERMEDIATE SURFACE

The rolling conditions between the C_1 and C_2 centrodes, respectively C_2 and C_3 are defined:

$$\delta = R_{r\Sigma} \cdot \varphi_1; \delta = R_{rS} \cdot \varphi_2. \tag{1}$$

For the surface to be generated, see Figure 1, we accept equations in form:

$$\Sigma: \begin{vmatrix} X = X(u); \\ Y = Y(u); \\ Z = p \cdot v, \end{vmatrix} \tag{2}$$

with p helical parameter of the surface to be generated.

2.1. Global movements determining absolute

The generating movements are defined in the rolling movements of the two centrodes: C_1, the center of the semi-finished product, this being a circle of radius $R_{r\Sigma}$ and C_2, the centrode of the rack, line η. The generating movements are:

- rotation of the blank product around the z-axis, angle φ_1, in relation to the xyz system:

$$x = \omega_3^T(\varphi_1) \cdot X \tag{3}$$

- the intermediate surface translation:

$$x = \xi + A, \mathrm{A} = \begin{pmatrix} -R_{r\Sigma} \\ \delta \\ 0 \end{pmatrix} \tag{4}$$

The specific enwrapping condition is determined by the "virtual pole" method. According to this method, the virtual pole represents the point of intersection between the normal to the profile Σ, drawn by the current point, and the centroid associated with this profile. If Willis's theorem is taken into account, according to which: "*two profiles transmitting rotational motion between two parallel axes admit, at the point of contact, a common normal passing through the gear pole*" for which the virtual pole overlaps the gear pole.

For this purpose, the vector normal to the profile Σ is defined as:

$$\vec{N}_\Sigma = \left(\lambda \cdot \dot{Y}_u\right) \cdot \vec{\imath} - \left(\lambda \cdot \dot{X}_u\right) \cdot \vec{\jmath} \tag{5}$$

with λ scalar variable parameter and $\dot{X}_u$, $\dot{Y}_u$, partial derivatives of equations (2) regarding the variable u.

If we consider that the position vector of the current point is:

$$\vec{r} = X(u) \cdot \vec{\imath} + Y(u) \cdot \vec{\jmath} \tag{6}$$

by summing the vectors (5) and (6) it is possible determining the position vector of the virtual gear pole in form:

$$\vec{N}_{Pv} = \vec{N}_\Sigma + \vec{r} = \left[X(u) + \lambda \cdot \dot{Y}_u\right] \cdot \vec{\imath} + \left[Y(u) - \lambda \cdot \dot{X}_u\right] \cdot \vec{\jmath} \tag{7}$$

The enwrapping condition can be determined by restriction that the virtual pole to belong to the C_1 centrode, $P_v \in C_1$. The equations of the centrode are:

$$C_1\colon \begin{vmatrix} X = R_{r\Sigma} \cdot \cos\varphi_1\,; \\ Y = -R_{r\Sigma} \cdot \sin\varphi_1\,, \end{vmatrix} \tag{8}$$

and the condition that the virtual pole to belong to the blank's centrode represents a dependency between independent variables u and φ_1:

$$\varphi_1 = \varphi_1(u) \tag{9}$$

2.2. Contact curve determining

The contact curve, as geometrical locus, in the global reference systems, where the piece's profile and the rack-gear tool's profile are tangents, is defined with the assembly of equations:

$$\begin{cases} x = \omega_3^T(\varphi_1) \cdot X, \\ \varphi_1 = \varphi_1(u). \end{cases} \tag{10}$$

2.3. Generating rack-gear profile determining

By transferring the contact curve in the space of rack-gear:

$$\xi = x + A \tag{11}$$

where the **A** matrix is given by equation (4), we can find the profile of the generating rack-gear's profile, S, in principle in form:

$$S\colon \begin{vmatrix} \xi = \xi(u); \\ \eta = \eta(u). \end{vmatrix} \tag{12}$$

Knowing the parametric equations of the rack-gear, a new contact curve can be defined between the rack and the future hob tool.

3. PERIPHERAL SURFACE OF THE FUTURE HOB TOOL

Figure 2 defines the reference systems: $x_0y_0z_0$ as a fixed reference system, with the y_0 axis superimposed on the η axis; $x_1y_1z_1$ - fixed reference system, with z_1 axis superimposed on the axis of the hob tool; $X_1Y_1Z_1$ - mobile reference system, integral with the primary peripheral surface of the screw tool; $\xi\eta\zeta$ mobile reference system, integral with the rack tool.

The kinematics of the generation process includes the translation of the rack (of the space $\xi\eta\zeta$), of the parameter δ and the rotation of the system $X_1Y_1Z_1$, of the plastic deformation tool, around the axis Z_1, of angle φ_2, opposite to φ_1, see Figure 1.

The complementary method of the "virtual pole" determined by the normal to the side of the rack (12), which intersects the centroid C2 in the virtual pole P_{v2}, is applied.

In this way are defined the directrix parameters of the normal $\vec{\boldsymbol{N}}_S$, with unitary vector:

$$\vec{n}_S = \dot{\eta}_u \cdot \vec{\imath} - \dot{\xi}_u \cdot \vec{\jmath} \tag{13}$$

Starting from the current point onto S surface the normal of this surface can be write as:

$$\vec{N}_S = [\xi(u) + \kappa \cdot \dot{\eta}_u] \cdot \vec{\imath} + [\eta(u) - \kappa \cdot \dot{\xi}_u] \cdot \vec{\jmath} \tag{14}$$

with κ variable scalar value, in the current point onto S.

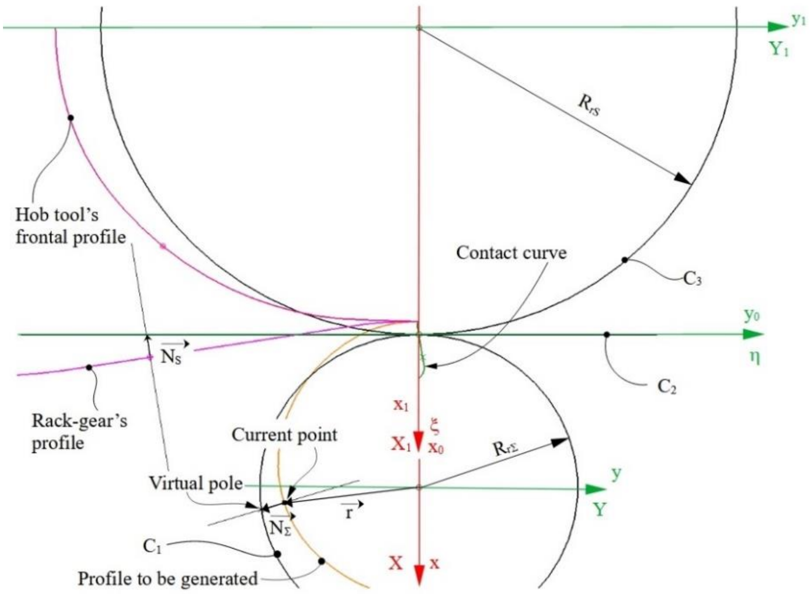


Fig. 2. Normal to the S intermediate surface; reference systems.

The C_2 centrode have equations:

$$C_2 = \begin{vmatrix} \xi = 0; \\ \eta = \delta_1, \end{vmatrix} \tag{15}$$

with δ_1 independent parameter in the translation movements along the η axis.

The conditions for determining the virtual pole onto C_2 are, from (14) and (15):

$$\begin{cases} \xi(u) + \kappa \cdot \dot{\eta}_u = 0; \\ \eta(u) - \kappa \cdot \dot{\xi}_u = \delta_1. \end{cases} \tag{16}$$

The enwrapping condition results form equations (16):

$$\kappa = \frac{-\xi(u)}{\dot{\eta}_u} = \frac{\eta(u) - \delta_1}{\dot{\xi}_u} \tag{17}$$

which allows determining the value δ_1, which determines the position of the virtual pole P_{v2}, in the enwrappinf condition between the rack-gear and the hob tool, so, the enwrapping condition between the intermediate surface and the cylindrical helical surface with constant pitch. The two worms have helical parameters: *p* for the generating surface and p_1 for the generated surface.

Note: If the helical surface Σ is a right worm, then the generating tool is a left worm with the same angle of the helix (Figure 3).

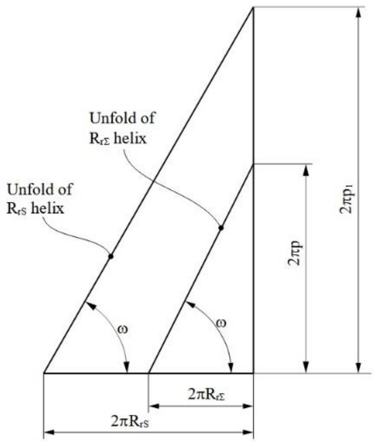


Fig. 3. Unfold of the helix onto the rolling cylinders $R_{r\Sigma}$ şi R_{rS}, of the blank, respectively of the hob tool.

The angle of the helix is:

$$\tan\omega = \frac{2\pi p}{2\pi R_{r\Sigma}} = \frac{2\pi p_1}{2\pi R_{rS}} \tag{18}$$

so,

$$\tan\omega = \frac{p}{R_{r\Sigma}} = \frac{p_1}{R_{rS}} \tag{19}$$

3.1. Contact curve with intermediate surface

The contact curve is defined by the set of equations:

- the absolute movement of the rack in relation to the fixed system, $x_0y_0z_0$:

$$x_0 = \xi + B, \mathrm{B} = \begin{pmatrix} 0 \\ \delta_1 \\ 0 \end{pmatrix} \tag{20}$$

- the enwrapping condition:

$$\xi(u)\cdot\dot{\xi}_u - [\eta(u) - \delta_1]\cdot\dot{\eta}_u = 0 \tag{21}$$

- the rolling condition of C_2 centrode onto C_3 centrode, this being the circle with radius R_{rS}:

$$\delta_1 = R_{rS}\cdot\varphi_2 \tag{22}$$

Also, the rolling condition of the two circular centrodes, C_1 with radius $R_{r\Sigma}$ and C_3 with radius R_{rS}, is:

$$R_{r\Sigma}\cdot\varphi_1 = R_{rS}\cdot\varphi_2 \Rightarrow \varphi_2 = i\cdot\varphi_1 \tag{23}$$

if we denote $\frac{R_{r\Sigma}}{R_{rS}=i}$.

3.2. The primary peripheral surface of the deforming screw tool

The surface of the deformation tool (helical surface of parameter p_1 and axis Z_1, (see Figure 1) is obtained by transposing the rack gear line, (20) and (21), in the system of the rack tool.

Define:
- position of fixed systems $x_0y_0z_0$ and $x_1y_1z_1$:

$$x_1 = x_0 + C, \mathrm{C} = \begin{pmatrix} R_{rS} \\ 0 \\ 0 \end{pmatrix} \tag{24}$$

- the rotation of the $X_1Y_1Z_1$ reference system around the Z_1 axis, the global motion:

$$x_1 = \omega_3^T(-\varphi_2) \cdot X_1 \tag{25}$$

- translation of the $\xi\eta\zeta$ reference system:

$$x_1 = \xi + B + C \tag{26}$$

which determine the transfer of the contact curve in the $X_1Y_1Z_1$ reference system of the hob tool:

$$\mathrm{X}_1 = \omega(\varphi_2) \cdot (\xi + \mathrm{B} + \mathrm{C}) \tag{27}$$

or, developed:

$$\begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} = \begin{pmatrix} \cos\varphi_2 & \sin\varphi_2 & 0 \\ -\sin\varphi_2 & \cos\varphi_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \xi(u) + R_{rS} \\ \eta(u) + \delta_1 \\ -p_1 \cdot \varphi_2 \end{pmatrix}. \tag{28}$$

Associated with the enwrapping condition (23) and the rolling condition (22), the (28) equations become:

$$S: \begin{vmatrix} X_1 = (\xi + R_{rS})\cos\varphi_2 + (\eta + \delta_1)\sin\varphi_2\,; \\ Y_1 = -(\xi + R_S)\sin\varphi_2 + (\eta + \delta_1)\cos\varphi_2\,; \\ Z_1 = -p_1 \cdot \varphi_2, \end{vmatrix} \tag{29}$$

representing the hob tool's primary peripheral surface of the counter-clock wise worm, with p_1 helical parameter, associated with the circular centrode with radius R_{rS}. Obviously, the, ξ and η coordinates are given by equations on form (12), the cylindrical surface with directrix parallel with ζ axis.

4. APPLICATIONS

4.1. Profiling of the generating tool for manufacturing worm shafts as part of the seat adjustment mechanism of some Audi cars

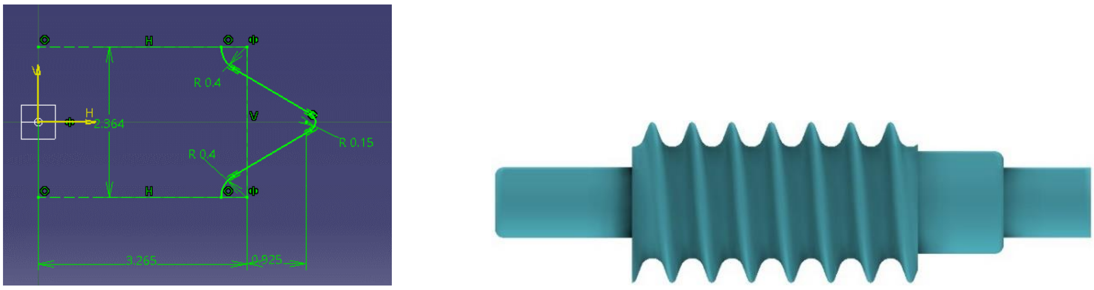
The worm shaft in the construction of an Audi car seat adjustment mechanism is shown in Figure 4.

a). b).

Fig. 4. a) The worm shaft from the construction of an Audi car seat adjustment mechanism; b) Numerical model obtained by scanning.

By measurement, using the GOM Inspect program, the dimensions of the axial section of the worm were determined and then its CAD model was made.

Figure 5.a. shows the profile of the screw-generating helix. Applying a helical motion to this profile, the three-dimensional numerical model of the worm is obtained, see Figure 5.b.



a). b).

Fig. 5. a) Axial profile of the worm shaft; b) 3D model of the worm shaft.

In order to determine the profile of the snail in the frontal plane, the intersection is made between the helical surface and a plane perpendicular to its axis. How to obtain this intersection is shown in Figure 6.

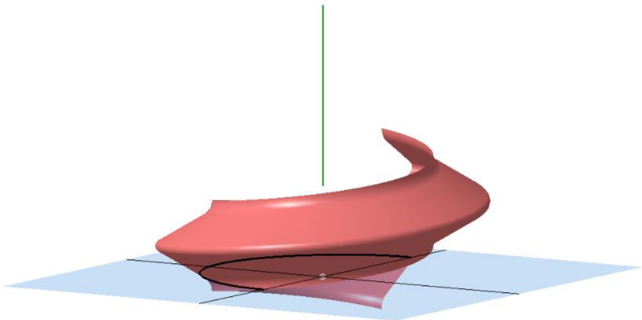


Fig. 6. Obtaining of the worm profile in the front plane.

Subsequently, on the respective intersection, which represents the frontal profile of the worm, in the frontal plane, a current point is chosen. At this point the normal is drawn to the curve and, using the command "*Intersection*" is determined the point where the normal intersects the centrode of the piece, i.e. the virtual pole (Figure 7).

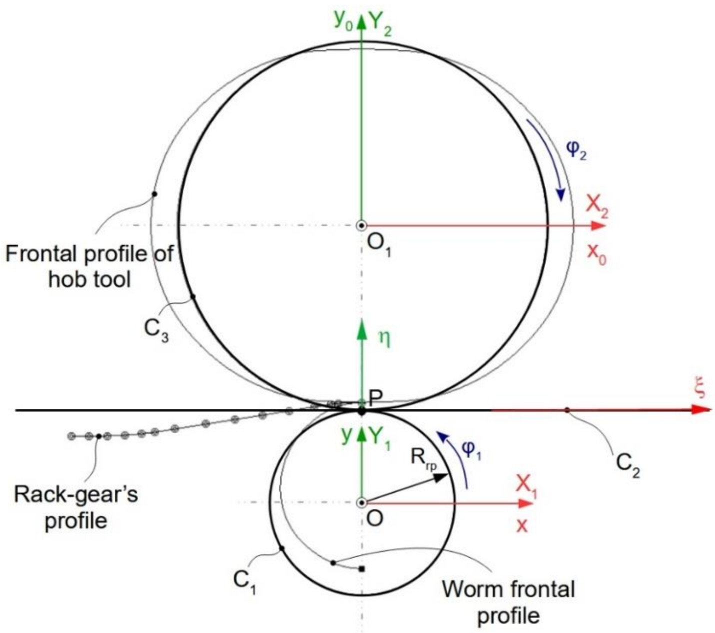


Fig. 7. Determining of the generating rack-gear profile and of the contact curve.

According to the algorithm presented in section 2, the angle φ_1 is determined, with which the part must be rotated for the virtual pole to overlap the gear pole. The current point is rotated by the determined angle and thus its position in the fixed space is obtained, according to equation (10). Moving by translation along the axis η the newly obtained point is determined the contact position in the space associated with the rack. The operation is resumed in reverse order, determining the position of the point of contact in the space of the worm tool, only this time the point is rotated by the angle φ_2. The value of the angle φ_2 is given by the running condition (23).

Repeating the presented algorithm, so that the current point covers the entire front profile of the generated worm, the front profile of the generating hob is obtained. This profile is then rotated about the axis of the tool, applying the movement (27), obtaining the primary peripheral surface of the tool, (29). Figure 8 shows the reciprocating surfaces Σ and S, for the worm shaft in the seat adjustment mechanism of some Audi cars, the axial profile of which is given in Figure 5.a.

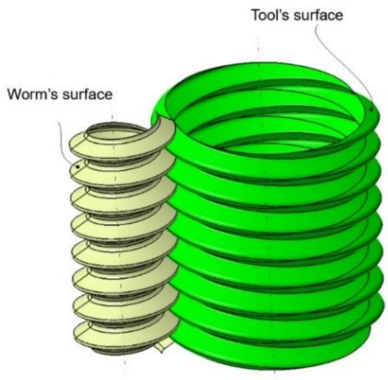


Fig. 8. Helical surface reciprocally enwrapped.

Table 1 shows the coordinates of the points belonging to the contact curve, the profile of the intermediate surface (generating rack-gear tool) and the front profile of the hob tool.

Table 1. Coordinates of points belong to the contact curve, the profile of the intermediate surface and the front profile of the hob tool.

Contact curve		Generating rack-gear		Hob profile	
x	y	ξ	η	X	Y
-4.340	0.000	-0.340	0.000	0.001	-9.135
-4.304	-0.024	-0.304	-1.033	0.877	-9.086
-4.271	-0.040	-0.271	-1.346	1.736	-8.941
-4.154	-0.025	-0.154	-2.079	2.561	-8.702
-3.980	0.003	0.020	-3.164	3.331	-8.376
-3.798	0.032	0.202	-4.299	3.887	-8.055
-3.608	0.063	0.392	-5.493	4.653	-7.479
-3.406	0.094	0.594	-6.757	5.702	-6.430
-3.193	0.128	0.807	-8.102	6.488	-5.323
-3.057	0.148	0.943	-8.955	7.045	-4.199
-2.987	0.099	1.013	-9.527	7.401	-3.090
-2.929	0.065	1.071	-10.270	7.583	-2.016
-2.892	0.041	1.108	-11.027	7.620	-1.295
-2.872	0.020	1.128	-11.794	7.632	-0.992
-2.865	0.000	1.135	-12.566	7.660	0.000

4.2. Profiling of the generating tool for machining worm shafts as part of the seat adjustment mechanism of some Mercedes cars

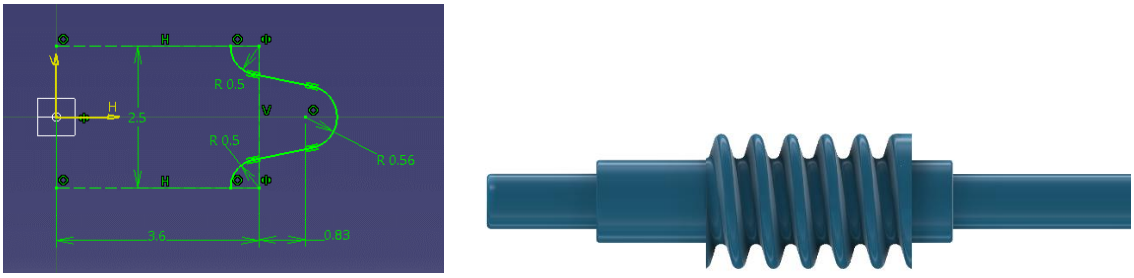
By measurement, the dimensions of the axial profile of the auger were determined using the program presented in subsection 4.1. Subsequently, its CAD model was developed.

The worm shaft in the construction of a seat adjustment mechanism for Mercedes cars is shown in Figure 9.

a). b).

Fig. 9. a) The worm shaft from the construction of a Mercedes car seat adjustment mechanism; b) Numerical model obtained by scanning.

Figure 10.a shows the profile of the spiral generator of the screw. Applying a helical motion to this profile, the three-dimensional numerical model of the snail is obtained, see figure 10.b.



a). b).

Fig. 10. a) Worm shaft axial profile; b) The three-dimensional model of the worm shaft.

Proceeding similarly to subsection 4.1, determine the profile of the generating rack and the contact curve, see Figure 11.

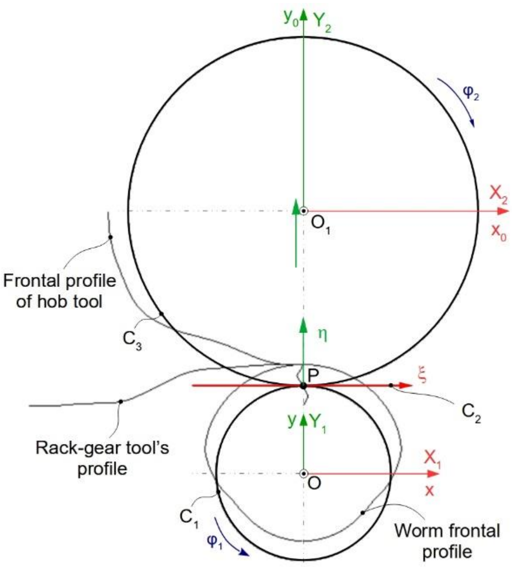


Fig. 11. Determining the profile of the generating rack and the contact curve.

Figure 12 shows the reciprocating surfaces *Σ* and *S*, for the worm in the seat adjustment mechanism of some Mercedes cars, the axial profile of which is given in Figure 10.a.

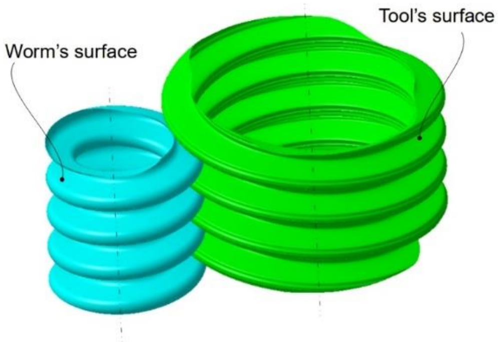


Fig. 12. Wrapped helical surfaces.

Table 2 shows the coordinates of the points belonging to the contact curve, the profile of the intermediate surface (generating rack) and the front profile of the worm.

Table 2. The coordinates of the points belonging to the contact curve, the profile of the intermediate surface and the front profile of the hob tool.

Contact curve		Generating rack-gear		Hob profile	
x	y	ξ	η	X	Y
-4.990	0.000	-0.990	0.000	7.010	0.000
-4.949	-0.038	-0.949	-2.115	6.805	-1.847
-4.796	-0.091	-0.796	-4.266	5.732	-4.797
-4.530	-0.267	-0.530	-5.555	5.477	-5.718
-4.082	-0.040	-0.082	-6.455	5.385	-5.961
-3.967	0.016	0.033	-6.689	5.277	-6.213
-3.727	0.132	0.273	-7.184	5.153	-6.474
-3.600	0.191	0.400	-7.447	5.009	-6.746
-3.474	0.206	0.526	-7.718	4.847	-7.017
-3.225	0.068	0.775	-9.247	3.532	-8.033
-3.129	0.031	0.871	-10.888	1.844	-8.678
-3.100	0.000	0.900	-12.566	0.000	-8.900
-4.990	0.000	-0.990	0.000	7.010	0.000

5. CONCLUSIONS

By applying the "virtual pole" method, the reciprocally enwrapping helical surfaces can be determined with other cylindrical helical surfaces of constant pitch.

The method involves determining the intermediate surface of the generating rack-gear tool which is mutually enwrapping with the front profile of the helical surface.

The three-dimensional graphical representation certifies the accuracy of the results obtained by applying the proposed method.

The method is simple to apply and allows both graphical and analytical application.

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