VARIANTS FOR EVALUATING THE RIGIDITY OF FLAT RING FLANGES

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Abstract: This paper describes an analytical way of comparative evaluation of the stiffnesses developed in assemblies with flat ring flanges, of optional type, welded to the cylindrical body of a pressure vessel. Based on the theory of the compatibility of deformations (radial displacements and rotations), the mathematical expressions necessary for the evaluation of the unitary radial bending moments and the unitary shearing forces of connection are established. With their help, the values of the rotation angles of the rings can be calculated and compared with the admissible ones. The present analysis considers the quantitative effect of the deformed gasket and the stiffness of the curved/bent screws on the tightness of the system. The methodology is flexible by introducing some selection factors, so that the mentioned influences can be easily separated and compared.

Keywords: assemblies with flat ring flanges, radial and angular deformations

1. INTRODUCTION

The complexity of industrial process equipment, used for the processing of various substances, has raised great problems in the way of scientific research, their design, manufacture and transportation, generally with large masses and dimensions. The configuration of the above problem includes dynamic or static seals (flat or necked ring flanges [1 – 13], respectively ring flanges with radial ribs [14 – 18], tightened with screws, clips or clamps [19, 20, 21], with flat or lenticular ring gaskets [22, 23], respectively without gaskets [24, 25].

Choosing the material of a sealing gasket is a difficult problem because it must meet a series of extremely important conditions such as: a) stability at operating temperature, pressure and chemical and mechanical aggressiveness of the working environment; c) to show resistance to friction and wear; e) to deform elasto-plastically, when the gasket is tightened, to completely fill the microasperities of the sealing surfaces.

The loss of tightness of a flanged assembly can occur if: a) there is no correct correlation between the state of the sealing surfaces and the tightness of the gasket during the assembly phase; b) the value of the remaining sealing pressure during exploitation is not higher than the value of the regime pressure; c) loosening or creep of the gasket and/or the screws (studs) is manifested; d) there are unwanted changes in the material structure of the assembly

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components and, in particular, the degradation of the gasket material. In this sense, the study of flange assemblies can be carried out taking into account static and/or dynamic loads, simple or combined (pressures, temperatures, transient regime, fatigue, creep, earthquake) [26-28], as well as corrosive aggressiveness and/or erosive for metallic construction materials or non-metallic (polymeric or composite). An essential role in confirming the load-bearing capacity of flanged assemblies is played by experimental works to determine resistance, rigidity and tightness. The established mathematical expressions clearly show the effect of the stiffness of the assembly on the tightness, which is why a smaller size of the ring diameter is required, simultaneously with the increase of its thickness, but also the increase of the number of tightening screws. All the previously mentioned elements have in mind, at the same time, ensuring a safe operation.

The topic presente in this paper is the way of evaluating the angular deformation of the ring of such flat ring flange under the action of static loads (internal pressure, temperature and gasket pressing pressure, friction developed when the gasket moves outward). The values obtained can determine the worst case situation in which constructive measures can be adopted to increase the rigidity of the construction.

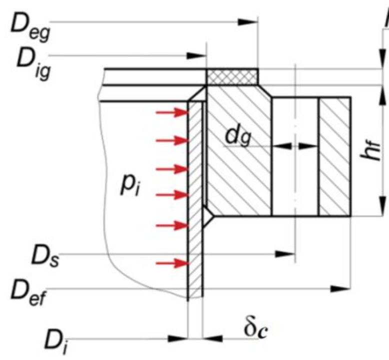

Fig. 1. Flat ring flanges, type A (dimensional characteristics – scheme) [13, 29].

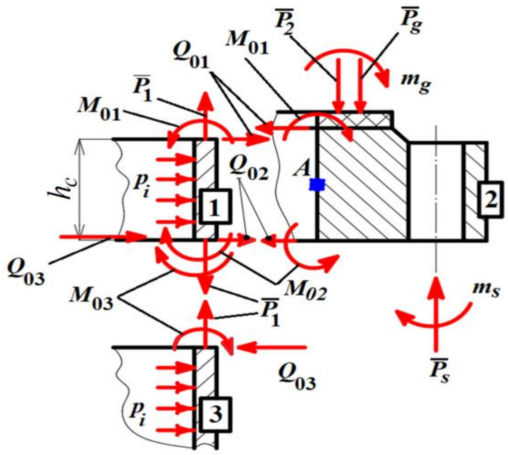

Fig. 2. Separation (hypothetical) of the elements of the assembly with flanges (scheme):
1 – cylindrical shell adjacent to the flange ring; 2 – flange ring; 3 – cylindrical shell (vessel body) [13, 29].

2. STUDY HYPOTHESES

In the following analysis the simplifying hypotheses specified in detail in the works [13, 29] are taken into account, of which the essential ones are noted:

1. The cylindrical shell adjacent to the flange ring (of constant thickness throughout the radial extension) is considered to have the length (Figure 1) $h_c < \ell_{sc} \approx 1.77 \cdot \sqrt{D_{mc} \cdot \delta_c}$ and constant thickness (h_c - the height of the wall of the cylindrical shell, adjacent to the flange ring, *mm; l_{sc}* - half wave length, *mm; $\delta_c -$* the wall thickness of the cylindrical shell, *mm*; $D_{mc} = D_i + \delta_c -$ the average diameter of the wall of the cylindrical shell, *mm*; $D_i -$ the inner diameter of the cylindrical shell, *mm*);

2. The effect of the welding cords for fixing the cylindrical ferrule with the flange ring on the general state of stress is neglected; the construction material of the cylindrical shell and the flange ring is considered homogeneous, continuous and isotropic, required in the elastic field;

3. Deformation of the rings in the radial direction and their rotation, according to the accepted hypothesis, is performed in a monobloc configuration; the effect of the screw holes on the rigidity of the flange ring is neglected;

4. The angle of rotation of the flange ring is conditioned according to the values of the internal connection loads ($M_{0j}^{(v)}, Q_{0j}^{(v)}$ (v - the study option $v = 1,2,3$; $j = 1,2,3$ - Figure 2), along with the effect of external loads, as well as the unitary resisting force manifested in the tendency to expel the gasket [13, 30], the unitary, **resistive bending moment**, m_g by the eccentric compression of the gasket, respectively the unitary, **resistant bending moment**, m_s**,** developed due to the curvature of the screws [14 -17]; all are related to the circumference accepted for the analysis (study variant); unitary cutting forces from the upper part $Q_{01}^{(v)}$ and from the lower part $Q_{02}^{(v)}$ of the flange ring is transferred to the middle level of the ring attaching the corresponding unit bending moments $0,5 \cdot h_f \cdot Q_{01}^{(v)}$ and $0,5 \cdot h_f \cdot Q_{02}^{(v)}$.

5. The present study considers three variants of analysis for evaluating the rotation of the flange ring:
- **Variant 1** – rotating the flange ring around its median circumference;
- **Variant 2** - rotating the flange ring around its inner (median) circumference;
- **Variant 3** - rotating the flange ring around the circumference of the centers of the screw holes (at the level of the middle surface of the ring);

6. Therefore, the expressions of the analyzed flange ring rotations are presented in the following forms:

Variant 1:

$$\vartheta^{(1)} = \frac{1}{2 \cdot k_{g1}} \cdot \left[\begin{array}{c} h_f \cdot Q_{01}^{(1)} - 2 \cdot \dfrac{D_{if}}{D_{mf}} \cdot M_{01}^{(1)} - h_f \cdot Q_{02}^{(1)} + 2 \cdot \dfrac{D_{if}}{D_{mf}} \cdot M_{02}^{(1)} - \\ - \left(\begin{array}{c} D_{mf} - D_2 - \\ -\mu_{fg} \cdot h_f \cdot c_s^{fg} \end{array} \right) \cdot \overline{P}_2 + \left(D_s - D_{mf} \right) \cdot \overline{P}_s - \left(\begin{array}{c} D_{mf} - D_{mg} + \\ +\mu_{fg} \cdot h_f \cdot c_s^{fg} \end{array} \right) \cdot \overline{P}_g \end{array} \right]; \qquad (1)$$

$$k_{g1} = \frac{E_f \cdot h_f^3 \cdot \ln\left(D_{ef} / D_{if} \right)}{6 \cdot D_{mg}} + \frac{D_{mg}}{D_{mf}} \cdot \frac{E_g \cdot c_g^3 \cdot \left(D_{eg} - D_{ig} \right)^3}{48 \cdot h_g} \cdot c_\vartheta^g + \frac{E_s \cdot n_s \cdot d_s^4 \cdot k_\vartheta^s}{16 \cdot l_s \cdot D_{mf}} \cdot c_\vartheta^s; \qquad (2)$$

Variant 2:

$$\vartheta^{(2)} = \frac{1}{2 \cdot k_{g2}} \cdot \left[\begin{array}{c} h_f \cdot Q_{01}^{(2)} - 2 \cdot M_{01}^{(2)} - h_f \cdot Q_{02}^{(2)} + M_{02}^{(2)} - \left(\begin{array}{c} D_2 - D_{if} - \\ -\mu_{fg} \cdot h_f \cdot c_s^{fg} \end{array} \right) \cdot \overline{P}_2 + \\ + \left(D_s - D_{if} \right) \cdot \overline{P}_s - \left(D_{mg} - D_{if} + \mu_{fg} \cdot h_f \cdot c_s^{fg} \right) \cdot \overline{P}_g \end{array} \right]; \qquad (3)$$

$$\begin{array}{c} k_{g2} = 2 \cdot \left(1 - \nu_f^2 \right) \cdot \left[\left(\dfrac{D_{ef}}{D_{if}} \right)^2 - 1 \right] \cdot \Re_f \cdot \dfrac{D_{if}}{D_{ef}^2} + \dfrac{D_{mg}}{D_{if}} \cdot \dfrac{E_g \cdot c_g^3 \cdot \left(D_{eg} - D_{ig} \right)^3 \cdot c_\vartheta^g}{48 \cdot h_g} + \\ + E_s \cdot n_s \cdot d_s^4 \cdot k_\vartheta^s \cdot c_\vartheta^s \Big/ \left(16 \cdot l_s \cdot D_{if} \right); \end{array} \qquad (4)$$

Varianta 3:

$$\vartheta^{(3)} = \frac{1}{2 \cdot k_{g3}} \cdot \left[\begin{array}{l} h_f \cdot \dfrac{D_{if}}{D_s} \cdot Q_{01}^{(1)} - 2 \cdot \dfrac{D_{if}}{D_s} \cdot M_{01}^{(1)} - h_f \cdot \dfrac{D_{if}}{D_s} \cdot Q_{02}^{(1)} + 2 \cdot \dfrac{D_{if}}{D_s} \cdot M_{02}^{(1)} + \\ + \left(D_s - D_2 - \mu_{fg} \cdot h_f \cdot c_s^{fg} \right) \cdot \overline{P}_2 + \left(D_s - D_{mg} + \mu_{fg} \cdot h_f \cdot c_s^{fg} \right) \cdot \overline{P}_g \end{array} \right]; \qquad (5)$$

$$k_{g3} = \frac{1}{k_{f3}} + \frac{n_s \cdot E_s \cdot d_s^4 \cdot c_\vartheta^s}{16 \cdot \ell_s \cdot D_s} + \frac{E_g \cdot c_g^3 \cdot D_{mg} \cdot \left(D_{eg} - D_{ig}\right)^3 \cdot c_\vartheta^g}{48 \cdot h_g \cdot D_s}; \qquad (6)$$

$$k_{f3} = D_s^2 \cdot \frac{1 - \mu_f + \left(1 + \mu_f\right) \cdot \alpha_{Df3}}{2 \cdot \alpha_{Df3} \cdot \left(\alpha_{Df3}^2 - 1\right) \cdot \left(1 - \mu_f^2\right) \cdot \Re_f \cdot D_{if}}; \; \alpha_{Df3} = D_s / D_{if}. \qquad (7)$$

The following notations were used in the previous equations: $D_{ef}, D_{if}, D_{eg}, D_{ig}, D_{mf}, D_{mg}, D_s -$ the inner diameter of the flange ring, respectively the outer one; the outer diameter of the gasket for sealing, respectively the inner one; the average diameter of the flange ring; the average diameter of the gasket for sealing; the diameter of the circumference of the centers of the screw holes *mm*; E_c, E_f, E_g, E_s - the modules of longitudinal elasticity of the materials of the cylindrical shell, of the flange ring, of the sealing gasket, respectively of the screws material, *N/mm²*; $\bar{F}_{fg} = \mu_{fg} \cdot \left(\bar{P}_g + \bar{P}_2\right)$ - unitary friction force between flange ring and sealing gasket, *N/mm*; P_s - the force developed in the screws of the flange assembly, in different operating conditions, *N*; $\bar{P}_s$ - the existing unit force on the diameter circumference D_s, *N/mm*; P_2 - the force developed by the internal pressure of the working environment, between the outer diameter of the cylindrical shell and the average diameter of the seal, *N*; $\bar{P}_2 -$ the existing unitary force on the area between the outer diameter of the cylindrical shell and the average diameter of the gasket, *N/mm*; $P_g -$ the force developed across the width of the seal gasket, *N*; $\bar{P}_g -$ the unitary force developed across the width of the seal gasket, *N/mm*; $\Re_f -$ cylindrical bending stiffness/rigidy of pressure vessel body, *N/mm*; c_g - the "reduction" coefficient of the initial gasket width, which can be chosen considering recommendations from the literature; c_ϑ^g, c_ϑ^s, c_ϑ^{fg} - factors that allow to easily determine the influences of "*resistant*" loads on the deformation of the flange ring (when the mentioned factors take values equal to unity, the influences are present; when the values are zero, the respective influences disappear); d_s - the calculated diameter of the joint screws, *mm;* h_f, h_g - the thickness of the flange ring and the thickness of the gasket for sealing, *mm;* k_{g1}, k_{g2}, k_{g3} - values dependent on the influence of unit bending moments produced by the presence of the flange ring, the screws or the gasket compressed eccentrically, *1/N*; l_s - the calculation length of the screws, *mm;* n_s - the number of assembly screws; μ_f - the coefficient of transverse contraction (Poisson) of the material of the flange ring; μ_{fg} - coefficient of friction between the gasket for sealing and the flange ring; $\vartheta^{(v)} -$ the angle of rotation of the flange ring $\left(v = 1, 2, 3\right)$, calculated in the adopted study version (it is easy to see that the maximum value of the angle of rotation of the flange ring is when $c_\vartheta^s = c_\vartheta^g = 0$, that is, when the effects of unit bending moments are neglected m_s and m_g).

3. ANALYTICAL STUDY. CONNECTION LOADS

To obtain the expressions of the unitary bending moments of the connection $M_{0j}^{(v)}$ and unitary forces of connection $Q_{0j}^{(v)}$, the assembly is broken-down into its component elements (Figure 2). In this sense, the compatibility equations of the deformations (radial displacements and rotations) between the previously mentioned elements are written: 1 – 2 (up), 1 – 2 (down) 1 (down) – 3, resulting in the algebraic system written in the form:

$$\left[A^{(v)}\right] \cdot \left\{S_l^{(v)}\right\} = \left\{T_l^{(v)}\right\}, \qquad (8)$$

in which:

$$\left[A^{(v)}\right]=\begin{bmatrix} a_{11}^{(v)} & a_{12}^{(v)} & \dots & a_{16}^{(v)} \\ a_{21}^{(v)} & a_{22}^{(v)} & \dots & a_{26}^{(v)} \\ \dots & \dots & \dots & \dots \\ a_{61}^{(v)} & a_{62}^{(v)} & \dots & a_{66}^{(v)} \end{bmatrix}, \tag{9}$$

represents the matrix of *influencing factors* $a_{ij}^{(v)}$ ($v=1,2,3;\ i=1,\cdots,6;\ \ j=1,\cdots,6$); the transposed vector of the connection load:

$$\left\{S_l^{(v)}\right\}=\left\{Q_{01}^{(v)}\quad M_{01}^{(v)}\quad Q_{02}^{(v)}\quad M_{02}^{(v)}\quad Q_{03}^{(v)}\quad M_{03}^{(v)}\right\}^T; \tag{10}$$

transposed vector of free terms (radial displacements and rotations under the action of external loads - pressure, temperature) - $b_j^{(v)}$ ($v=1,2,3;\ j=1,\cdots,6$):

$$\left\{T_l^{(v)}\right\}=\left\{b_1^{(v)}\quad b_2^{(v)}\quad \dots\quad b_6^{(v)}\right\}^T. \tag{11}$$

From equality (8) the way of evaluating the values of the unknowns of the present problem - unitary shear forces $Q_{oj}^{(v)}$ ($j=1,2,3;\ v=1,2,3$) and unitary bending moments $M_{0j}^{(v)}$($j=1,2,3;\ v=1,2,3$) is deduced- written in the form:

$$\left\{S_l^{(v)}\right\}=\left[A^{(v)}\right]^{-1}\cdot\left\{T_l^{(v)}\right\}, \tag{12}$$

where $\left[A^{(v)}\right]^{-1}$ represents the inverse of the $\left[A^{(v)}\right]$ matrix, whose determinant has a value other than zero.

The expressions of the influencing factors, different for the three study variants, are presented in the following forms:

Variant 1:

$$\begin{gathered} a_{11}^{(1)}= f_{1q}\Big/\left(2\cdot k_c^3\cdot\Re_c\right)-k_{wf}-h_f^2\cdot D_{if}\Big/\left(4\cdot k_{g1}\cdot D_{mf}\right);\ a_{12}^{(1)}=f_{1m}\Big/\left(2\cdot k_c^2\cdot\Re_c\right)+ \\ +h_f\cdot D_{if}\Big/\left(2\cdot k_{g1}\cdot D_{mf}\right);\ a_{13}^{(1)}=f_{qd}\left(h_c\right)\Big/\left(2\cdot k_c^3\cdot\Re_c\right)-k_{wf}+h_f^2\cdot D_{if}\Big/\left(4\cdot k_{g1}\cdot D_{mf}\right); \\ a_{14}^{(1)}=f_{md}\left(h_c\right)\Big/\left(2\cdot k_c^2\cdot\Re_c\right)-h_f\cdot D_{if}\Big/\left(2\cdot k_{g1}\cdot D_{mf}\right);\ \ a_{21}^{(1)}=-f_{23q}\Big/\left(2\cdot k_c^2\cdot\Re_c\right)- \\ -h_f\cdot D_{if}\Big/\left(2\cdot k_{g1}\cdot D_{mf}\right);\ a_{22}^{(1)}=-f_{2m}\Big/\left(k_c\cdot\Re_c\right)+D_{if}\Big/\left(k_{g1}\cdot D_{mf}\right); \\ a_{23}^{(1)}=f_{qr}\left(h_c\right)\Big/\left(2\cdot k_c^2\cdot\Re_c\right)+h_f\cdot D_{if}\Big/\left(2\cdot k_{g1}\cdot D_{mf}\right);\ a_{24}^{(1)}=f_{mr}\left(h_c\right)\Big/\left(2\cdot k_c\cdot\Re_c\right)- \\ -D_{if}\Big/\left(k_{g1}\cdot D_{mf}\right);\ \ a_{31}^{(1)}=f_{qd}\left(h_c\right)\Big/\left(2\cdot k_c^3\cdot\Re_c\right)-k_{wf}+h_f^2\cdot D_{if}\Big/\left(4\cdot k_{g1}\cdot D_{mf}\right); \\ a_{32}^{(1)}=f_{md}\left(h_c\right)\Big/\left(2\cdot k_c^2\cdot\Re_c\right)-h_f\cdot D_{if}\Big/\left(2\cdot k_{g1}\cdot D_{mf}\right);\ \ a_{33}^{(1)}=f_{1q}\Big/\left(2\cdot k_c^3\cdot\Re_c\right)-k_{wf}- \\ -h_f^2\cdot D_{if}\Big/\left(4\cdot k_{g1}\cdot D_{mg}\right);\ a_{34}^{(1)}=f_{1m}\Big/\left(2\cdot k_c^2\cdot\Re_c\right)+h_f\cdot D_{if}\Big/\left(2\cdot k_{g1}\cdot D_{mf}\right); \\ a_{41}^{(1)}=-f_{qr}\left(h_c\right)\Big/\left(2\cdot k_c^2\cdot\Re_c\right)-h_f\cdot D_{if}\Big/\left(2\cdot k_{g1}\cdot D_{mf}\right);\ a_{42}^{(1)}=-f_{mr}\left(h_c\right)\Big/\left(2\cdot k_c\cdot\Re_c\right)+ \\ +D_{if}\Big/\left(k_{g1}\cdot D_{mf}\right);\ a_{43}^{(1)}=f_{23q}\Big/\left(2\cdot k_c^2\cdot\Re_c\right)+h_f\cdot D_{if}\Big/\left(2\cdot k_{g1}\cdot D_{mf}\right); \\ a_{44}^{(1)}=f_{2m}\Big/\left(k_c\cdot\Re_c\right)-D_{if}\Big/\left(k_{g1}\cdot D_{mf}\right). \end{gathered} \tag{13}$$

Variant 2:

$$a_{11}^{(2)} = f_{1q}/\left(2\cdot k_c^3\cdot\Re_c\right) - k_{wf} - h_f^2/\left(4\cdot k_{g2}\right);\quad a_{12}^{(2)} = f_{1m}/\left(2\cdot k_c^2\cdot\Re_c\right) + h_f/\left(2\cdot k_{g2}\right);$$
$$a_{13}^{(2)} = f_{qd}\left(h_c\right)/\left(2\cdot k_c^3\cdot\Re_c\right) - k_{wf} + h_f^2/\left(4\cdot k_{g2}\right);$$
$$a_{14}^{(2)} = f_{md}\left(h_c\right)/\left(2\cdot k_c^2\cdot\Re_c\right) + +h_f/\left(2\cdot k_{g2}\right);$$
$$a_{21}^{(2)} = -f_{23q}/\left(2\cdot k_c^2\cdot\Re_c\right) - h_f/\left(2\cdot k_{g2}\right);$$
$$a_{22}^{(2)} = -f_{2m}/\left(k_c\cdot\Re_c\right) + 1/k_{g2};$$
$$a_{23}^{(2)} = f_{qr}\left(h_c\right)/\left(2\cdot k_c^2\cdot\Re_c\right) + h_f/\left(2\cdot k_{g2}\right);$$
$$a_{24}^{(2)} = f_{mr}\left(h_c\right)/\left(2\cdot k_c\cdot\Re_c\right) - 1/k_{g2};$$
$$a_{31}^{(2)} = f_{qd}\left(h_c\right)/\left(2\cdot k_c^3\cdot\Re_c\right) - k_{wf} - h_f^2/\left(4\cdot k_{g2}\right);$$
$$a_{32}^{(2)} = f_{md}\left(h_c\right)/\left(2\cdot k_c^2\cdot\Re_c\right) + h_f/\left(2\cdot k_{g2}\right);$$
$$a_{33}^{(2)} = f_{1q}/\left(2\cdot k_c^3\cdot\Re_c\right) - k_{wf} + h_f^2/\left(4\cdot k_{g2}\right);\ a_{34}^{(2)} = f_{1m}/\left(2\cdot k_c^2\cdot\Re_c\right) + h_f/\left(2\cdot k_{g2}\right);$$
$$a_{41}^{(2)} = -f_{qr}\left(h_c\right)/\left(2\cdot k_c^2\cdot\Re_c\right) - h_f/\left(2\cdot k_{g2}\right);\quad a_{42}^{(2)} = -f_{mr}\left(h_c\right)/\left(2\cdot k_c\cdot\Re_c\right) + 1/k_{g2};$$
$$a_{43}^{(2)} = f_{23q}/\left(2\cdot k_c^2\cdot\Re_c\right) + h_f/\left(2\cdot k_{g2}\right);\quad a_{44}^{(2)} = f_{2m}/\left(k_c\cdot\Re_c\right) - 1/k_{g2}. \tag{14}$$

Variant 3:

$$a_{11}^{(3)} = f_{1q}/\left(2\cdot k_c^3\cdot\Re_c\right) - k_{wf} - h_f^2\cdot D_{if}/\left(4\cdot k_{g3}\cdot D_s\right);\quad a_{12}^{(3)} = f_{1m}/\left(2\cdot k_c^2\cdot\Re_c\right) +$$
$$+ h_f\cdot D_{if}/\left(2\cdot k_{g3}\cdot D_s\right);\ a_{13}^{(3)} = f_{qd}\left(h_c\right)/\left(2\cdot k_c^3\cdot\Re_c\right) - k_{wf} + h_f^2\cdot D_{if}/\left(4\cdot k_{g3}\cdot D_s\right);$$
$$a_{14}^{(3)} = f_{md}\left(h_c\right)/\left(2\cdot k_c^2\cdot\Re_c\right) - h_f\cdot D_{if}/\left(2\cdot k_{g3}\cdot D_s\right);\quad a_{21}^{(3)} = -f_{23q}/\left(2\cdot k_c^2\cdot\Re_c\right) -$$
$$- h_f\cdot D_{if}/\left(2\cdot k_{g3}\cdot D_s\right);\ a_{22}^{(3)} = -f_{2m}/\left(k_c\cdot\Re_c\right) + D_{if}/\left(k_{g3}\cdot D_s\right);$$
$$a_{23}^{(3)} = f_{qr}\left(h_c\right)/\left(2\cdot k_c^2\cdot\Re_c\right) + h_f\cdot D_{if}/\left(2\cdot k_{g3}\cdot D_s\right);\quad a_{24}^{(3)} = f_{mr}\left(h_c\right)/\left(2\cdot k_c\cdot\Re_c\right) -$$
$$- D_{if}/\left(k_{g3}\cdot D_s\right);\ a_{31}^{(3)} = f_{qd}\left(h_c\right)/\left(2\cdot k_c^3\cdot\Re_c\right) + h_f\cdot D_{if}/\left(4\cdot k_{g3}\cdot D_s\right);$$
$$a_{32}^{(3)} = f_{md}\left(h_c\right)/\left(2\cdot k_c^2\cdot\Re_c\right) - h_f\cdot D_{if}/\left(2\cdot k_{g3}\cdot D_s\right);\quad a_{33}^{(3)} = f_{1q}/\left(2\cdot k_c^3\cdot\Re_c\right) - k_{wf} -$$
$$- h_f^2\cdot D_{if}/\left(4\cdot k_{g3}\cdot D_s\right);\ a_{34}^{(3)} = f_{1m}/\left(2\cdot k_c^2\cdot\Re_c\right) + h_f\cdot D_{if}/\left(2\cdot k_{g3}\cdot D_s\right);$$
$$a_{41}^{(3)} = f_{qr}\left(h_c\right)/\left(2\cdot k_c^2\cdot\Re_c\right) - h_f\cdot D_{if}/\left(2\cdot k_{g3}\cdot D_s\right);\ a_{42}^{(3)} = -f_{mr}\left(h_c\right)/\left(2\cdot k_c\cdot\Re_c\right) +$$
$$+ D_{if}/\left(k_{g3}\cdot D_s\right);\ a_{43}^{(3)} = -f_{23q}/\left(2\cdot k_c^2\cdot\Re_c\right) + h_f\cdot D_{if}/\left(2\cdot k_{g3}\cdot D_s\right);$$
$$a_{44}^{(3)} = f_{2m}/\left(k_c\cdot\Re_c\right) - D_{if}/\left(k_{g3}\cdot D_s\right). \tag{15}$$

The similar forms of the expressions of the influence factors, for the three comparative variants ($v = 1, 2, 3$), have the configuration:

$$a_{15}^{(v)} = f_{qd}\left(h_c\right)/\left(2\cdot k_c^3\cdot\Re_c\right);\ a_{16}^{(v)} = f_{md}\left(h_c\right)/\left(2\cdot k_c^2\cdot\Re_c\right);\ a_{25}^{(v)} = f_{qr}\left(h_c\right)/\left(2\cdot k_c^2\cdot\Re_c\right);$$
$$a_{26}^{(v)} = f_{mr}\left(h_c\right)/\left(2\cdot k_c\cdot\Re_c\right);\ a_{35}^{(v)} = f_{1q}/\left(2\cdot k_c^3\cdot\Re_c\right);\ a_{36}^{(v)} = f_{1m}/\left(2\cdot k_c^2\cdot\Re_c\right);$$
$$a_{45}^{(v)} = f_{23q}/\left(2\cdot k_c^2\cdot\Re_c\right);\ a_{46}^{(v)} = f_{2m}/\left(k_c\cdot\Re_c\right);\ a_{51}^{(v)} = f_{qd}\left(h_c\right)/\left(2\cdot k_c^3\cdot\Re_c\right);$$

$$a_{52}^{(v)} = f_{md}(h_c) / (2 \cdot k_c^2 \cdot \Re_c);\ a_{53}^{(v)} = f_{1q} / (2 \cdot k_c^3 \cdot \Re_c);\ a_{54}^{(v)} = f_{1m} / (2 \cdot k_c^2 \cdot \Re_c);$$
$$a_{55}^{(v)} = (f_{1q} - 1) / (2 \cdot k_c^3 \cdot \Re_c);\ a_{56}^{(v)} = (f_{1m} + 1) / (2 \cdot k_c^2 \cdot \Re_c);\ a_{61}^{(v)} = -f_{qr}(h_c) / (2 \cdot k_c^2 \cdot \Re_c);$$
$$a_{62}^{(v)} = -f_{mr}(h_c) / (2 \cdot k_c \cdot \Re_c);\ a_{63}^{(v)} = f_{23q} / (2 \cdot k_c^2 \cdot \Re_c);\ a_{64}^{(v)} = f_{2m} / (k_c \cdot \Re_c);$$
$$a_{65}^{(v)} = (1 + f_{23q}) / (2 \cdot k_c^2 \cdot \Re_c);\ a_{66}^{(v)} = (1 + f_{2m}) / (k_c \cdot \Re_c). \qquad (16)$$

On the other hand, the relations for estimating the free terms (radial displacements - b_1, b_3, b_5 and rotations - b_2, b_4, b_6) are used expressions:

Variant 1:

$$b_1^{(1)} = b_3^{(1)} = \frac{p_i}{8 \cdot k_c^2 \cdot \Re_c} \cdot \left[2 - \frac{\mu_c \cdot D_i^2}{D_{mc}^2}\right] + 0{,}5 \cdot \alpha_c \cdot D_{mc} \cdot \Delta T_c + \frac{h_f}{4 \cdot k_{g1}} \cdot \left[-\begin{pmatrix} D_{mf} - D_2 - \\ -\mu_{fg} \cdot h_f \cdot c_{\vartheta}^{fg} \end{pmatrix} \cdot \overline{P}_2 + \right.$$
$$\left. + (D_s - D_{mf}) \cdot \overline{P}_s - (D_{mf} - D_{mg} + \mu_{fg} \cdot h_f \cdot c_{\vartheta}^{fg}) \cdot \overline{P}_g\right] - \Delta R(\Delta T_f)_{D = D_{if}};$$
$$b_2^{(1)} = b_4^{(1)} = \frac{1}{2 \cdot k_{g1}} \cdot \left[-\begin{pmatrix} D_{mf} - D_2 - \\ -\mu_{fg} \cdot h_f \cdot c_{\vartheta}^{fg} \end{pmatrix} \cdot \overline{P}_2 + (D_s - D_{mf}) \cdot \overline{P}_s - \begin{pmatrix} D_{mf} - D_{mg} + \\ + \mu_{fg} \cdot h_f \cdot c_{\vartheta}^{fg} \end{pmatrix} \cdot \overline{P}_g\right]; \qquad (17)$$

Variant 2:

$$b_1^{(2)} = b_3^{(2)} = \frac{p_i}{4 \cdot k_c^4 \cdot \Re_c} \cdot \left[1 - \frac{\mu_c \cdot D_i^2}{2 \cdot D_{mc}^2}\right] + 0{,}5 \cdot \alpha_c \cdot D_{mc} \cdot \Delta T_c + \frac{h_f}{4 \cdot k_{g2}} \cdot \left[-\begin{pmatrix} D_2 - D_{if} - \\ -\mu_{fg} \cdot h_f \cdot c_{\vartheta}^{fg} \end{pmatrix} \cdot \overline{P}_2 + \right.$$
$$\left. + (D_s - D_{if}) \cdot \overline{P}_s - (D_{mg} - D_{if} + \mu_{fg} \cdot h_f \cdot c_{\vartheta}^{fg}) \cdot \overline{P}_g\right] - \Delta R(\Delta T_f)_{D = D_{if}};$$
$$b_2^{(2)} = b_4^{(2)} = \frac{1}{2 \cdot k_{g2}} \cdot \left[-\begin{pmatrix} D_2 - D_{if} - \\ -\mu_{fg} \cdot h_f \cdot c_{\vartheta}^{fg} \end{pmatrix} \cdot \overline{P}_2 + (D_s - D_{if}) \cdot \overline{P}_s - \begin{pmatrix} D_{mg} - D_{if} + \\ + \mu_{fg} \cdot h_f \cdot c_{\vartheta}^{fg} \end{pmatrix} \cdot \overline{P}_g\right]. \qquad (18)$$

Variant 3:

$$b_1^{(3)} = b_3^{(3)} = \frac{p_i}{4 \cdot k_c^4 \cdot \Re_c} \cdot \left(1 - \frac{\mu_c \cdot D_i^2}{2 \cdot D_{mc}^2}\right) + 0{,}5 \cdot \alpha_c \cdot D_{mc} \cdot \Delta T_c + \frac{h_f}{4 \cdot k_{g3}} \cdot \left[\begin{pmatrix} D_s - D_2 - \\ -\mu_{fg} \cdot h_f \cdot c_{\vartheta}^{fg} \end{pmatrix} \cdot \overline{P}_2 + \right.$$
$$\left. + (D_s - D_{mg} + \mu_{fg} \cdot h_f \cdot c_{\vartheta}^{fg}) \cdot \overline{P}_g\right] - \Delta R(\Delta T_f)_{D = D_{if}};$$
$$b_2^{(3)} = b_4^{(3)} = \frac{h_f}{2 \cdot k_{g3}} \cdot \left[(D_s - D_2 - \mu_{fg} \cdot h_f \cdot c_{\vartheta}^{fg}) \cdot \overline{P}_2 + (D_s - D_{mg} + \mu_{fg} \cdot h_f \cdot c_{\vartheta}^{fg}) \cdot \overline{P}_g\right]. \qquad (19)$$

Therewith $b_5^{(1)} = b_5^{(2)} = b_5^{(3)} = 0,$ respectively $b^{(1)} = b^{(2)} = b^{(3)} = 0.$

In the above equations the auxiliary values (the corresponding geometric characteristics are noted in Figures 1 and 2) are found:

$$P_1 = 0.25 \cdot \pi \cdot D_{ec}^2 \cdot p_i;\ \overline{P}_1 = P_1 / (\pi \cdot D_{mc});\ P_2 = 0.25 \cdot \pi \cdot (D_{mg}^2 - D_{ec}^2) \cdot p_i;$$
$$\overline{P}_2 = 0.5 \cdot (D_{mg} - D_{ec}) \cdot p_i;\ P_g = 0.25 \cdot \pi \cdot (D_{eg}^2 - D_{ig}^2) \cdot p_{sg};\ D_{ec} = D_i + 2 \cdot \delta;$$

$$\bar{P}_g = 0.5\cdot\left(D_{eg}-D_{ig}\right)\cdot p_{sg}\,;\ \Re_c = E_c\cdot\delta^3 \big/ \sqrt{12\cdot\left(1-\mu_c^2\right)}\,;\ k_c = \sqrt[4]{12\cdot\left(1-\mu_c^2\right)} \Big/ \sqrt{D_{mc}\cdot\delta}\,;$$

$$D_{mc}=D_i+\delta\,;\ D_{mg}=0.5\cdot\left(D_{ig}+D_{eg}\right);\ D_{mf}=0.5\cdot\left(D_{if}+D_{ef}\right);$$

$$k_{wf}=0.5\cdot D_{if}\cdot\left[\left(1-\mu_f\right)\cdot D_{ef}^2-\left(1-2\cdot\mu_f\right)\cdot D_{if}^2\right] \Big/ \left[h_f\cdot E_f\cdot\left(D_{ef}^2-D_{if}^2\right)\right];$$

$$s=sin\left(k_c\cdot h_c\right);\ c=cos\left(k_c\cdot h_c\right);\ s_h=sh\left(k_c\cdot h_c\right)=0{,}5\cdot\left[exp\left(k_c\cdot h_c\right)-exp\left(-k_c\cdot h_c\right)\right];$$

$$c_h=ch\left(k_c\cdot h_c\right)=0{,}5\cdot\left[exp\left(k_c\cdot h_c\right)+exp\left(-k_c\cdot h_c\right)\right];$$

$$N=N\left(k_c\cdot h_c\right)=sh^2\left(k_c\cdot h_c\right)-sin^2\left(k_c\cdot h_c\right);$$

$$f_{1m}=f_{1m}\left(k_c\cdot h_c\right)=\left(s_h^2+s^2\right)\big/N;\ f_{2m}=f_{2m}\left(k_c\cdot h_c\right)=-\left(s\cdot c+s_h\cdot c_h\right)\big/N;$$

$$f_{1q}=f_{1q}\left(k_c\cdot h_c\right)=\left(s\cdot c-s_h\cdot c_h\right)\big/N;\ f_{2q}=f_{2q}\left(k_c\cdot h_c\right)=s_h^2\big/N;$$

$$f_{3q}=f_{3q}\left(k_c\cdot h_c\right)=s^2\big/N;\ f_{23q}=f_{23q}\left(k_c\cdot h_c\right)=\left(s^2-s_h^2\right)\big/N;$$

$$f_{md}=f_{md}\left(k_c\cdot h_c\right)=f_{1m}\cdot c\cdot c_h+f_{2m}\cdot\left(c\cdot s_h+s\cdot c_h\right)+s\cdot s_h\,;$$

$$f_{mr}=f_{mr}\left(k_c\cdot h_c\right)=f_{1m}\cdot\left(c\cdot s_h-s\cdot c_h\right)+2\cdot f_{2m}\cdot c\cdot c_h+c\cdot s_h+s\cdot c_h\,;$$

$$f_{qr}=f_{qr}\left(k_c\cdot h_c\right)=f_{1q}\cdot\left(c\cdot s_h-s\cdot c_h\right)+f_{2q}\cdot\left(c\cdot c_h-s\cdot s_h\right)+f_{3q}\cdot\left(c\cdot c_h+s\cdot s_h\right). \qquad (19)$$

In relations (13) to (19), the notations were also used: $\Re_c -$ cylindrical bending stiffness/rigidity of the shell wall, N/mm; $k_c -$ stress mitigation factor ,$1/mm$; $\Delta R\left(\Delta T_f\right)_{D=D_{if}}$ - the radial displacement of the flange ring under the influence of the thermal gradient, for the accepted law for temperature variation [31]; $\Delta T_c, \Delta T_f$ - the thermal gradient of the vessel body, related to the temperature of the external environment, respectively the thermal gradient of the flange ring at the level of its inner surface, *K*; $f_{1m},\ f_{2m},\ f_{md}\left(h_c\right), f_{mr}\left(h_c\right),\ f_{1q},\ f_{2q},\ f_{23q},\ f_{qd}\left(h_c\right), f_{qr}\left(h_c\right)$ - influencing factors of connection loads (unit shear forces and unit bending moments) [32]; $p_i -$ the internal pressure of the working environment, N/mm^2; $p_{sg} -$ compression pressure of the gasket, accepted depending on its material and the recommendations of the recognized standards, N/mm^2; k_{wf} -influence factor on the width of the flange ring of the unitary shear forces (connection), mm^2/N; $\alpha_c, \alpha_f -$ thermal deformation factors characteristics of the cylindrical shell and the flange ring, K^{-1}; $\mu_c -$ the coefficient of transverse contraction (Poisson) of the material of the cylindrical shell.

4. CONCLUSIONS

This paper takes into discussion the evaluation of the deformation of the ring of a ring flat flange, welded to the cylindrical shell. The rotation of the ring is considered around some circumferences placed in the medial plane of the ring, in three variants: a) at the median surface (characteristic of the middle radius of the ring); b) on the inner surface; c) at the level of the centers of the screw holes. Static loads that act are taken into account: the internal pressure of the working environment and the temperatures developed in operation characteristic of the median surface of the cylindrical wall along the radius of the ring (according to a specific law [31]). The material of the cylindrical shell and that of the ring (same or different from the ferrule) is isotropic, continuous and homogeneous.

The mechanical and thermal loads can be evaluated analytically by means of the compatibility theory between the elastic deformations produced in the component elements. In this sense, the three elements of the structure (Figure 2) are distinguished, used for writing a linear algebraic system. By solving it, the expressions of the connection loads (unit radial bending moments and unit shear forces) are deduced. If necessary, the loading scheme can be corrected and the axial and annular stresses developed on the inner and outer surface of the cylindrical shell, along it, can be calculated. In this way, it is possible to establish the position of the plane where the equivalent stress is maximum and, obviously, below the value of the admissible resistance of the construction material, under the operating conditions.

A suitable calculation computation programme can lead to the optimization of the construction, with the minimization of material consumption and ensuring a safe operation. The previously exposed methodology allows, at the same time, an evaluation of the stress by using discrete values of the external loads in the case of a transient regime of the pressure, respectively the temperature. The exposure has in its composition the presence of factors that allow the deduction of a maximum deformation, appreciating the sealing of the construction. Values equal to unity or zero give the mentioned appreciation. By canceling the internal pressure and the working temperature, the established relationships allow the assessment of the deformation of the flange ring under tightening conditions (mounting).

The results obtained through the methodology presented in the article can be developed in the framework of additional research on the behavior of flanged assemblies in areas such as creep, fatigue, the effect of residual stresses in weld seams or the prevention of cracking.

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