

RESERVE ESTIMATION FOR AN AQUIFER-SUPPORTED RESERVOIR USING MATERIAL BALANCE

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Abstract: This study uses a straight-line material balance method to estimate the reserves of an undersaturated reservoir supported by an aquifer. Some computer algorithms to implement Van-Everdingen and Hurst models (VEHM) to calculate cumulative water influx were envisaged through the study's developed tool "QUANTIFY". The tool was then tested using published data. The Original Oil in Place (OOIP) for the given data was estimated to be 312.32 MMSTB with an R² value of 0.99483. Subsequently, a comparative analysis was performed on the results obtained from MBAL and QUANTIFY, using Dake's result as a base/correct case. It was estimated that OOIP for QUANTIFY had a 0.1% error rate, while MBAL, a commercial software used to verify the result, had a 1.2% error rate. Invariably, the percentage errors were lower for the QUANTIFY software than for MBAL. The 3D plot of the reservoir energy drive from the "QUANTIFY" software, which was used to visualize the reservoir energy qualitatively, shows that the reservoir was mostly driven by water influx and fluid expansion. This study has demonstrated that "Water Influx" parameters can be computed appropriately without the use of charts or tables. The study has strengthened the validity of the Van Everdingen and Hurst aquifer model for reserve estimation for an aquifer-supported reservoir.

Keywords: reservoir, QUANTIFY, MBAL, simulation, water influx, undersaturated

1. INTRODUCTION

The material balance method is very important for estimating original oil in place (OOIP), predicting reservoir performance, and determining distinct contributions from different drive mechanisms, particularly for conventional reservoirs [1]. In 2017, Molokwu and Onyekonwu [2] identified the material balance approach as one of the methods used by petroleum engineers for interpreting and predicting reservoir performance. They also described the material balance model as a zero-dimensional model that considers the reservoir as a tank having the same pressure and PVT properties throughout a particular time [2]. The general material balance equations originally presented by Schilthuis [3] in 1936 were based on hydrocarbon pore volumes. The conventional material balance comprises three variables: average reservoir pressure, recovery factor, and cumulative gas-oil ratio (GOR). With a known average pressure, other parameters, such as compressibility and fluid PVT properties, can be easily determined. Universal-type curve and straight-line analysis have been used to estimate original gas in place (OGIP) and other reservoir properties [4]. However, reservoir estimation is crucial in reserve management, exploitation, exploration, and production [5]. In reserve estimation, the goal is to principally evaluate a reservoir to estimate and assess the stock oil initially in place (STOIP) and analyze the reservoir's past and present performance [6]. To improve the clarity of these estimations, petroleum engineers often employ several reserve estimation techniques. However, these techniques depend on the quality and maturity of the available data. More so, the

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extent and nature of the commercially recoverable hydrocarbons from the subsurface cannot be determined with a high degree of precision because recoveries from subsurface reservoirs depend largely on the heterogeneities of the reservoir rock and the type of reservoir drive mechanism [1].

Natural water drive reservoirs are bounded on a portion or all of their peripheries by water-bearing rocks called aquifers [7]. The aquifers may be so large compared to the reservoir they adjoin as to appear infinite for all practical purposes. On the other hand, the aquifer itself may be entirely bounded by impermeable rock, so the reservoir and aquifer form a closed (volumetric) unit. When discussing water influx into a reservoir, it is common to speak of edge or bottom water. Bottom water occurs directly beneath the oil, and edge water occurs off the structure's flanks at the oil's edge as distinct from water injection, which has already been qualitatively [8]. Water influx can also be referred to as water encroachment or aquifer influx. It can be defined as an underground layer of water-bearing porous rock which flows out into any available space in the reservoir rock [9-10]. In this context, an aquifer is referred to as a large pool of water body underlying a hydrocarbon accumulation in the reservoir structure that is made up of more than one fluid arranged according to density differences [9].

The estimation process becomes complicated when employing the material balance approach in determining STOOIP for an aquifer-supported reservoir. It thus would require using aquifer models such as Van –Everdingen and Hurst, Carter Tracy, and Fetkovich, among others, for history matching. Before using these models (especially Van-Everdingen and Hurst), dimensionless times and water influx need to be computed to specify the cumulative water influx to account for the influence of the underground aquifer. So far, calculating dimensionless water influxes is usually performed using standard plots, which makes the processing time-consuming [11–12]. Previous research works like those performed by Omoniyi and Adeolu [1] in 2014 that determined STOOIP for aquifer-supported reservoirs using MBE had assumed water influx data to be known, which is often not the case in reality. Also, in 2021, Omonusi and Okologume [13] performed a study where they computed the reserve estimate for an undersaturated reservoir using a least squares regression approach with a material balance equation. However, an aquifer-supported reservoir was not accounted for in their study. During the analysis process, various graphs are frequently used to characterize the conceptual model of the reservoir-aquifer system [14]. Nonetheless, this study seeks to employ material balance expressed as a straight line to quantify reserves for an undersaturated reservoir with a strong aquifer. A stochastic analytical and computer model was developed to describe the reservoir's reserve estimate, as mentioned earlier, adequately.

Therefore, the objectives of this study was to use an efficient computational approach to determine Stock Tank Oil Originally in Place (STOOIP) for an aquifer-supported reservoir. Consequently, the study developed a novel computational approach for calculating cumulative water influx using the Van-Everdingen and Hurst superposition theorem. The study uses the complex polynomial method to directly calculate dimensionless water influx for both finitely and infinitely acting reservoirs. Subsequently, it uses the Odeh and Havlena material balance approach to finding the STOOIP for an undersaturated water drive reservoir.

2. EXPERIMENTAL SETUP

2.1. Mathematical models

In order, to quantify reserves for an aquifer-supported undersaturated reservoir, it is necessary first to specify appropriate aquifer models that best describe water influx into the reservoir. There are several aquifer models, among which Van Everdingen and Hurst Model (VEHM) offers a more realistic approach since it considers pressure drop in the reservoir throughout its production life. Consequently, Van Everdingen and Hurst Models are usually employed to match real-life history data in the petroleum industry, especially before reserve volume estimation. However, VEHM is often difficult to apply because it operates with the principle of superposition. Therefore, coding the procedure in computer programs becomes difficult. This study uses complex polynomials, as presented by Klins's group [15], to calculate dimensionless water influx W_{eD} while considering different reservoir flow regimes (both infinitely and finitely acting reservoirs). The approach to estimating water encroachment is well suited for computer-based reservoir engineering studies, especially reservoir simulation, because a wider range of reservoir properties can be examined, and no time-consuming matrix-search techniques are used.

2.1.1. Cross-over point t_{cross}

All aquifers act as if they are infinite for small values of the dimensionless time. However, at later times, boundary effects are felt, and finite aquifer behaviour deviates accordingly. The cross-over point at a particular dimensionless

radius (aquifer-to-reservoir radius ratio) refers to the dimensionless time (t_D) at which boundary effects are felt. Once this cross-over value of t_D is determined, the user can decide whether the finite or infinite set of polynomials is appropriate to calculate dimensionless water influx W_{eD} .

$$t_{cross} = -1.767 - 0.606(r_D) + 0.12368(r_D)^{2.25} + 3.02[\ln(r_D)]^{0.5} \quad (1)$$

For values of $t_D < t_{cross}$, the aquifer is infinite-acting; thus, the infinite-aquifer approach discussed in subsequent sections should be used. If otherwise, that is $t_D \geq t_{cross}$, then the polynomial for finitely-acting aquifer would be used. More so, dimensionless time is calculated as follows:

$$t_D = \frac{2.309kt}{\mu\phi c_t r_o^2} \quad (2)$$

where, t are time in years, μ is *viscosity*, c_t is total compressibility, ϕ is *porosity*, r_o is reservoir outer radius.

2.1.2. Determination of dimensionless water influx, W_{eD}

a) Finite aquifers:

Van –Everdingen and Hurst model is given by [13]:

$$W_{eD}(t_D) = \frac{r_D^2 - 1}{2} - 2 \sum_{n=1}^{\infty} \frac{e^{-\alpha_n^2 t_D} J_1^2(\alpha_n r_D)}{\alpha_n^2 [J_0^2(\alpha_n) - J_1^2(\alpha_n r_D)]} \quad (3)$$

where, t_D refers to the dimensionless time and is shown in equation (2), r_D is the ratio of the aquifer radius to the reservoir radius (r_e/r_w), J_1 and J_0 refer to the Bessel function of order 1 and 0, respectively. While α defines the roots of the following equation:

$$J_1(\alpha_n r_D) Y_0(\alpha_n) - Y_1(\alpha_n r_D) J_0(\alpha_n) = 0 \quad (4)$$

where J_1 and Y_1 are Bessel functions of order 1. However, equation (3) was expressed in a polynomial form as follows:

$$W_{eD}(t_D) = \frac{r_D^2 - 1}{2} - \frac{2e^{-\alpha_1^2 t_D} J_1^2(\alpha_1 r_D)}{\alpha_1^2 [J_0^2(\alpha_1) - J_1^2(\alpha_1 r_D)]} - \frac{2e^{-\alpha_2^2 t_D} J_1^2(\alpha_2 r_D)}{\alpha_2^2 [J_0^2(\alpha_2) - J_1^2(\alpha_2 r_D)]} \quad (5)$$

where:

$$\alpha_1 = -0.00222107 - 0.627638 \operatorname{csch}(r_D) + 6.2777915(r_D)^{-2.734405} + 1.2708(r_D)^{-1.100417} \quad (6)$$

and

$$\alpha_2 = -0.00796608 - 1.85408 \operatorname{csch}(r_D) + 18.71169(r_D)^{-2.758326} + 4.829162(r_D)^{-1.009021} \quad (7)$$

$\operatorname{csch}(x)$ refers to the hyperbolic cosecant function, which is computed as follows:

$$\operatorname{csch}(x) = \frac{1}{e^x - e^{-x}} \quad (8)$$

Also, the first-order Bessel functions are computed as shown in equation (9) and equation (10).

At condition: $0 \leq x < 3.0$.

$$J_1(x) = \left[0.5 - 0.56249985 \left(\frac{x}{3}\right)^2 + 0.21093573 \left(\frac{x}{3}\right)^4 - 0.03954289 \left(\frac{x}{3}\right)^6 + 0.00443319 \left(\frac{x}{3}\right)^8 - 0.00031761 \left(\frac{x}{3}\right)^{10} + 0.00001109 \left(\frac{x}{3}\right)^{12} \right] x \quad (9)$$

At condition: $3.0 \leq x < \infty$.

$$J_1(x) = (x)^{-0.5} F_1(\cos \theta_1) \quad (10)$$

where:

$$F_1 = b_0 + b_1 \left(\frac{3}{x}\right) + b_2 \left(\frac{3}{x}\right)^2 + b_3 \left(\frac{3}{x}\right)^3 + b_4 \left(\frac{3}{x}\right)^4 + b_5 \left(\frac{3}{x}\right)^5 + b_6 \left(\frac{3}{x}\right)^6 \quad (11)$$

$b_0 = 0.79788456$, $b_1 = 0.00000156$, $b_2 = 0.01659667$, $b_3 = 0.00017105$, $b_4 = -0.00249511$, $b_5 = 0.00113653$, $b_6 = -0.00020033$.

$$\theta_1 = x - 2.35619449 + 0.12499612 \left(\frac{3}{x}\right) + 0.00005650 \left(\frac{3}{x}\right)^2 - 0.00637879 \left(\frac{3}{x}\right)^3 + 0.00074348 \left(\frac{3}{x}\right)^4 + 0.0079824 \left(\frac{3}{x}\right)^5 - 0.00029166 \left(\frac{3}{x}\right)^6 \quad (12)$$

b) For Infinitely – acting aquifers ($t_D < t_{cross}$): for infinite aquifers, the value of W_{eD} as a function of dimensionless time is determined by Van Everdingen and Hurst as follows:

$$W_{eD} = \frac{4}{\pi^2} \int_0^\infty \frac{(1-e^{-u^2 t_D}) du}{u^3 [J_0^2(u) + Y_0^2(u)]} \quad (13)$$

An analytical solution to this integral is not available, and numerical methods are difficult to use near the origin because of the asymptotic nature of the function. For evaluation, the integral was broken into two parts such that equation (13) becomes:

$$W_{eD} = \frac{4}{\pi^2} \int_0^\delta \frac{(1-e^{-u^2 t_D}) du}{u^3 [J_0^2(u) + Y_0^2(u)]} + \frac{4}{\pi^2} \int_\delta^\infty \frac{(1-e^{-u^2 t_D}) du}{u^3 [J_0^2(u) + Y_0^2(u)]} \quad (14)$$

Again, in the paper presented by Klins's group [15], equation (14) was solved analytically using non-linear regression to obtain a set of the polynomial as shown in the following equations:

Condition: At $t_D \leq 0.01$,

$$W_{eD} = \frac{2}{\sqrt{\pi}} \sqrt{t_D} \quad (15)$$

Condition: At $0.01 \leq t_D < 200$,

$$W_{eD} = \frac{1.129552(t_D)^{0.5002034} + 1.160436t_D + 0.2642821(t_D)^{1.5} + 0.01131791(t_D)^{1.979139}}{1 + 0.5900113(t_D)^{0.5002034} + 0.04589742(t_D)} \quad (16)$$

Condition: At $200 \leq t_D < 2 \times 10^{12}$

$$W_{eD} = 10^{[4.3989 + 0.43693 \times \ln t_D - 4.16078(\ln t_D)^{0.09}]} \quad (17)$$

2.1.3. Linear expression of MBE using regression analysis

In this study, VEHM was used to estimate the value of cumulative water influx through superposition. A least square regression model is required to compute the best fit curve. The regression formulas are expressed as follows:

$$\frac{F}{E_0} = N + a_1 \frac{W_e}{E_0} \quad (18)$$

Thus, to evaluate the value of a_1 in equation (18), which is the slope of the equation, this study fitted a linear least square regression model with the assumption that the intercept equals N. Therefore, a_1 becomes:

$$a_1 = \frac{i \sum \left(\frac{F}{E_0} \times \frac{W_e}{E_0} \right) - \sum \frac{F}{E_0} \times \sum \frac{W_e}{E_0}}{I \sum \left(\frac{W_e}{E_0} \right)^2 - \left(\sum \frac{W_e}{E_0} \right)^2} \quad (19)$$

N= intercept – becomes the stock tank oil originally in place (STOOIP).

2.2. Model assumptions

The assumptions made for the mathematical models that were employed in the quantification of the oil reserve are stated below:

- i. The reservoir is considered to be a tank.
- ii. Pressure, temperature, and rock and fluid properties are not space dependent.
- iii. Uniform hydrocarbon saturation and pressure distribution (homogenous reservoir).
- iv. Thermodynamic equilibrium is always attained.
- v. Isothermal condition apply.
- vi. Production data is reliable.
- vii. The reservoir is a water drive one.
- viii. It has a water influx.
- ix. Rock and fluid expansion was assumed to be negligible and was not accounted for.

The above assumptions were made using the Van Everdingen and Hurst aquifer models that were simplified into a set of polynomials [15]. Also, in previous research regarding dimensionless pressure and pressure derivatives, as presented in the literature, numerical integration was used to compute P_D and P'_D . However, this procedure is often complex and difficult to use near the origin because of the asymptotic nature of the functions involved. Thus, this study employed a simpler approach using a set of polynomials that were easier to implement than numerical methods. More so, to apply these equations (polynomials) in the calculation of dimensionless water influx, it is necessary to create a computer program. The computer program made it easy to quantify oil reserves with a strong aquifer, which is the major objective of this study. The algorithms of the developed computer program and the pseudocode descriptions, are presented in subsequent sections and subsections.

2.3. Computer model (QUANTIFY) description

The computer model (QUANTIFY) developed in this study is a reservoir engineering toolkit for calculating oil reserves for an undersaturated oil reservoir with water influx. The mathematical models discussed in the previous sections were incorporated into the developed toolkit "QUANTIFY". The toolkit, however, was developed to decide the appropriate correlation/polynomial to use depending on the conditions of t_D and the cross-over point. The correlations to calculate dimensionless water influx differ since the supposed aquifer may be acting finitely or infinitely. At a specified dimensionless time, t_D , boundary effects can either be felt or may not be felt (because the pressure disturbance has reached the boundary).

Even so, the software is intelligent enough to know the two conditions. The software was developed using Microsoft Visual C# (a programming language built on Microsoft's dot net framework). The splash screen of the developed software "QUANTIFY" is shown in Figure 1.

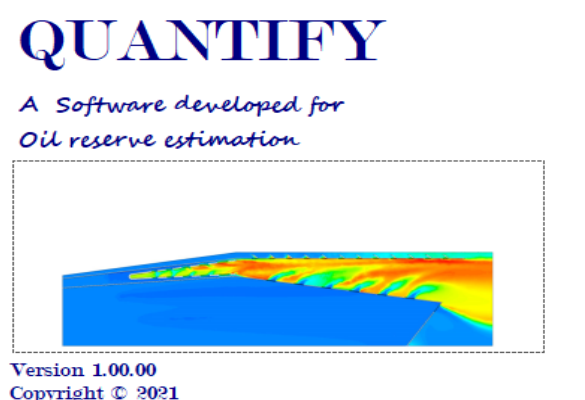


Fig. 1. Software splash screen.

2.3.1. Computer model development

This section discusses the algorithms and presents the pseudocode descriptions of the developed software "QUANTIFY". First, all necessary functions were created in a class. These functions include:

- i. The hyperbolic cosecant function ($\text{csch}(x)$).
- ii. Bessel function of order 1 ($J_1(x)$).
- iii. Dimensionless water influx function $We_D(t_D)$.

A computer superposition algorithm was developed to calculate dimensionless water influx prior to reserve estimation. The steps in using the developed software “QUANTIFY” is described as follows:

1. Launch the QUANTIFY software and select “match aquifer model” in the “Begin modelling” tab of the main form as shown on the main form illustrated in Figure 2.
2. In the dialog form that appears, fill the reservoir rock and fluid properties like rock compressibility, outer/inner radius, porosity etc. and click on “Import” button to select an excel pressure data file (see Figure 3).
3. Click on “Calculate” button to calculate dimensionless time (t_D) and dimensionless water influx (W_D).
4. Click on “Superimpose” to employ the superposition algorithm in calculating for water influx in barrels. Then click on “Done” to exit form.
5. On the main form, select “match pvt data” in the “Begin modelling” tab. A dialog form appears.
6. Import PVT data from excel .csv). Fit aquifer model, calculate regression and perform history matching to display calculated stock tank oil originally in place.
7. Click on “energy plot” button to display the dive mechanism of the reservoir as shown in Figure 4.
8. If the calculated R^2 value is far from one, repeat step 2 to 6, using a different outer/inner radius (r_{eD}). Otherwise, if R^2 value is very close to 1, the estimated STOOIP becomes the true value.

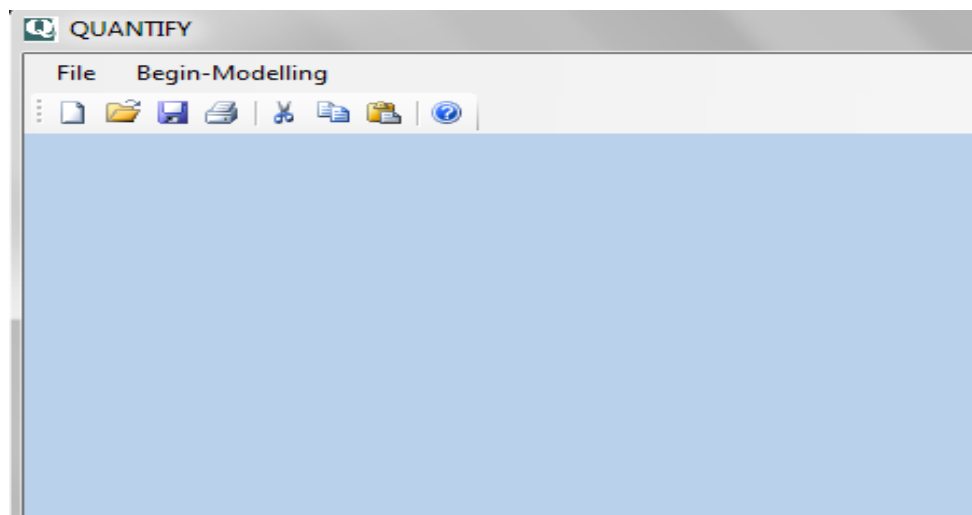


Fig. 2. Main form of QUANTIFY software.

Reservoir Acqifer Parameters

ReD	5.1
porosity	0.25
K(md)	200
Cf	0.000004
Cw	0.000003
viscosity	0.55
h(ft)	100
Bw	1
Encroachment angle	140
Reservoir radius	9200

	T (yrs)	ΔP	tD(red)	Wd(tD)	We(MMbbbls)
▶ 1		120			
2		225			
3		196			
4		170			
5		146			
6		123			
7		105			
8		84			
9		64			
10		47			

Fig. 3. Match aquifer model dialog.

MATCH PVT DATA

IMPORT DATA		FIT-AQUIFER		CALCULATE REGRESSION		HISTORY MATCH		ENERGY PLOT	
T(yrs)	Np(MMstb)	Rp(Scf/stb)	Bo(rb/stb)	Rs(Scf/stb)	Bg(scF/stb)	We(MMbbbl)	F(MMbbbl)	Eo	
0	0	650	1.404	650	0.00093				
1	7.88	760	1.374	592	0.00098				
2	18.42	845	1.349	545	0.00107				
3	29.15	920	1.329	507	0.00117				
4	40.69	975	1.316	471	0.00128				
5	50.14	1025	1.303	442	0.00139				
6	58.42	1065	1.294	418	0.0015				
7	65.39	1095	1.287	398	0.0016				
8	70.74	1120	1.28	383	0.0017				
9	74.54	1145	1.276	381	0.00176				
10	77.43	1160	1.273	364	0.00182				

Fig. 4. Match PVT history data.

The input data adapted from Dake [11-12] and are illustrated in Table 1 to Table 5.

Table 1. Reservoir fluid properties data [11-12].

GOR (Rs)	650
Oil Gravity	40
(Yg)	0.7
Salinity	14000

Table 2. PVT history data [11-12].

Time (year)	Pressure (psia)	Solution GOR (scf/STB)	Oil FVF (rb/STB)	Gas FVF (rb/STB)	Oil Viscosity (cp)	Gas Viscosity (cp)
0	2740	650	1.404	0.00093	0.54	0.0148
1	2500	592	1.374	0.00098	0.589	0.01497
2	2290	545	1.349	0.00107	0.518	0.01497
3	2109	507	1.329	0.00117	0.497	0.01497
4	1949	471	1.316	0.00128	0.497	0.01497
5	1818	442	1.303	0.00139	0.497	0.01497
6	1702	418	1.294	0.00150	0.497	0.01497
7	1608	398	1.287	0.00160	0.497	0.01497
8	1535	383	1.280	0.00170	0.497	0.01497
9	1480	381	1.276	0.00176	0.497	0.01497
10	1440	364	1.273	0.00182	0.497	0.00182

Table 3. Reservoir and Aquifer data [11-12].

Aquifer data		Reservoir data	
Parameter	Value	Parameter	Value
Reservoir thickness	100	Temperature	115
Reservoir radius	9200	Initial pressure	2740
Aquifer radius	46000	Porosity	0.25
Encroachment angle	140	Swc	0.05

Aquifer permeability	200	Cw	3.00E-06
		Cf	4.00E-06

Table 4. Relative permeability data [11-12].

	Residual sat	Endpoint	Exponent
K_{rw}	0.25	0.039336	0.064557
K_{ro}	0.15	0.8	10.5533
K_{rg}	0.05	0.9	1

Table 5. Production history data [11-12].

Time (dd/mm/ yyyy)	Reservoir Pressure (psia)	Cum oil Produced (MMSTB)	Cum Gas Produced (MMSCF)	Cum Water Produced (MMSTB)
1/8/1994	2740	0	0	0
1/8/1995	2500	7.88	5988.8	0
1/8/1996	2290	18.42	15564.9	0
1/8/1997	2109	29.15	26818	0
1/8/1998	1949	40.69	39672.8	0
1/8/1999	1818	50.14	51393.5	0
1/8/2000	1702	58.42	62217.3	0
1/8/2001	1608	65.39	71602.8	0
1/8/2002	1535	70.74	79228.8	0
1/8/2003	1480	74.54	85348.3	0
1/8/2004	1440	77.43	89818.8	0

3. RESULTS AND DISCUSSION

The results obtained from the developed software “QUANTIFY” are presented in this section and discussed herein. Dimensionless water influx and time were first determined from the pressure production data. After this, the Van- Everdingen and Hurst superposition method calculated cumulative water influx. Aquifer and reservoir data were then matched to have a perfectly fitted curve in total compliance with the conventional Odeh and Havlena material balance plot. Subsequently, the least square regression was done to quantify the reserve for an undersaturated aquifer-supported reservoir. Figure 5 presents the result of water influx calculations using QUANTIFY, and the subsequent results in Figure 6 to Figure 9 are also obtained from the QUANTIFY tool developed in this study.

T (yrs)	ΔP	tD(red)	Wd(tD)	We(MMbbbls)
1	120	5.668622492819...	4.9105	3.79818650372
2	225	11.33724498563...	7.5529	12.963636726131
3	196	17.00586747845...	9.2758	24.33219432920...
4	170	22.67448997127...	10.3993	36.41894281033...
5	146	28.34311246409...	11.1319	48.308201836406
6	123	34.01173495691...	11.6097	59.42734931764...
7	105	39.68035744973...	11.9212	69.55759150882...
8	84	45.34897994255...	12.1243	78.44317959786...
9	64	51.01760243537...	12.2567	85.95984328554...
10	47	56.68622492819...	12.3431	92.117973021998

Fig. 5. Water influx calculations using QUANTIFY.

In order to fit an aquifer model to the production history data, Van-Everdingen and Hurst model was employed. Correlations and polynomials [15] were used to determine the dimensionless time and dimensionless water influx. Also, an efficient superposition computer algorithm was developed to calculate the water influx in barrels from available data. The calculated variables are illustrated in Figure 5. After that, PVT data was input into the QUANTIFY tool, and MBE variables were determined, as shown in Figure 6.

HISTORY MATCH		ENERGY PLOT			
B _g (scf/stb)	W _e (MMbbl)	F(MMbbl)	E _o	F/E _o (MMstb)	W _e /E _o
0.00093					
0.00098	3.79818650372	12.1244832	0.026840000000...	451.7318628912...	141.5121648181...
0.00107	12.963636726131	30.7614	0.057350000000...	536.3801220575...	226.0442323649...
0.00117	24.33219432920...	52.8259215	0.092310000000...	572.26650958726	263.5921820952...
0.00128	36.41894281033...	79.7979728	0.14112	565.4618253968...	258.0707398691...
0.00139	48.308201836406	105.9643718	0.18812	563.2807346374...	256.7946089538...
0.0015	59.42734931764...	132.29209	0.238	555.8491176470...	249.6947540321...
0.0016	69.55759150882...	157.079858	0.2862	548.8464640111...	243.0384049923...
0.0017	78.44317959786...	179.177346	0.3299	543.1262382540...	237.7786589810...
0.00176	85.95984328554...	195.3425056	0.34544	565.4889578508...	248.8416028414...
0.00182	92.117973021998	210.7427796	0.38952	541.0319870609...	236.4909966676...

Fig. 6. Results for MBE variables calculation using QUANTIFY.

Also, regression analysis was performed to fit a straight-line curve to the production history data. Regression variables calculated by the developed tool are represented in Figure 7. Although the data used to validate the QUANTIFY tool was known to be predominantly water drive, it was still necessary to validate this information. To achieve this validation, the "developed tool" computed the Water Drive Index (WDI) and the Depletion Drive Index (DDI) for each time interval using the conventional material balance approach. The calculated variables for WDI and DDI are shown in Figure 7.

F/E _o (MMstb)	W _e /E _o	(F/E) _{est}	WDI	DDI
	0	312.3192626718...		
451.7318628912...	141.5121648181...	451.3397087123...	0.313265847382...	0.691381964231...
536.3801220575...	226.0442323649...	534.3833519933...	0.421425446375...	0.582272254001...
572.26650958726	263.5921820952...	571.2701647002...	0.460610882655...	0.545758414024...
565.4618253968...	258.0707398691...	565.8459429590...	0.456389323343...	0.552325990269...
563.2807346374...	256.7946089538...	564.59228190968	0.455890985015...	0.554464662940...
555.8491176470...	249.6947450321...	557.6174310161...	0.449213171533...	0.561877769985...
548.8464640111...	243.0384049923...	551.0782945220...	0.442816745536...	0.569046688192...
543.1262382540...	237.7786589810...	545.9111610993...	0.437796302652...	0.575039909093...
565.4889578508...	248.8416028414...	556.7793107878...	0.440046793817...	0.552299489381...
541.0319870609...	236.4909966676...	544.6461716944...	0.437110933038...	0.577265799695...

Fig. 7. Regression analysis results and energy drive variables determination.

However, a 3D visualisation plot was performed by QUANTIFY tool for clarity. The plot is represented in Figure 8. From the plot, it was observed that the reservoir is a combination of water and depletion drive energy. Furthermore, the plot showed a depletion drive between 69-55% and a water drive between 31-45 %. Thus, indicating that water-drive aquifer models can be fitted prior to reserve estimation.

Invariably, some runs were performed in this study using QUANTIFY software to obtain the STOOIP of the given reservoir at the highest R² value. These runs were done by varying the outer/inner reservoir radius (r_{eD}). Nonetheless, it was discovered that a r_{eD} value of 5.1 gave an OOIP of 312.32 MMSTB with an R² value of 0.99483 – which was the closest value to 1 (see Figure 9). It was then concluded that the r_{eD} value of 5.1 is the reservoir's true outer/inner radius because it matched perfectly with production history data and gave the highest R² value.

In a nutshell, the basic functionalities of the developed tool, QUANTIFY, are as follows:

1. It determines the dimensionless water influx parameters using the Klins et al. [15] polynomial method.
2. It adopts a novel superposition algorithm developed in this study for the determination of the cumulative water influx.

3. It employs the least square regression model to compute the STOOIP computationally without plotting charts. The charts presented by the tool were only for visual verification.
4. It can show the reservoir's drive mechanism in a 3D plot for the sake of clarity.

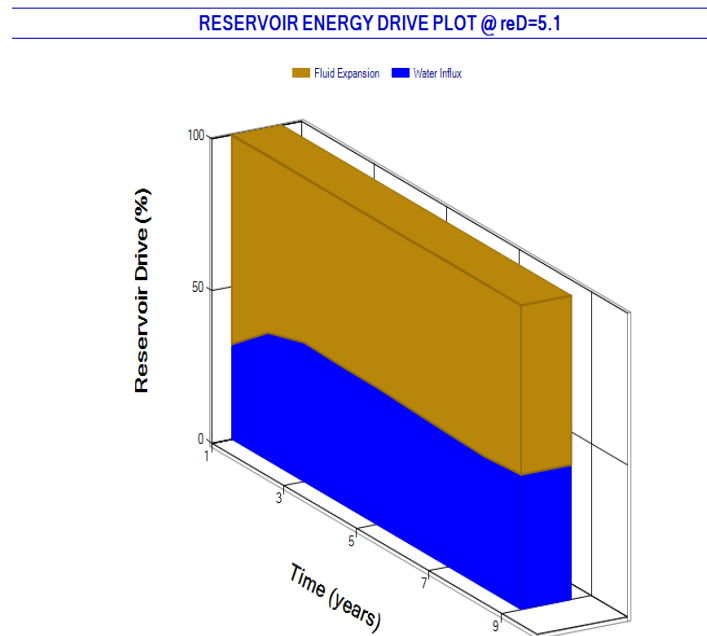


Fig. 8. Reservoir Energy drive plot using QUANTIFY.

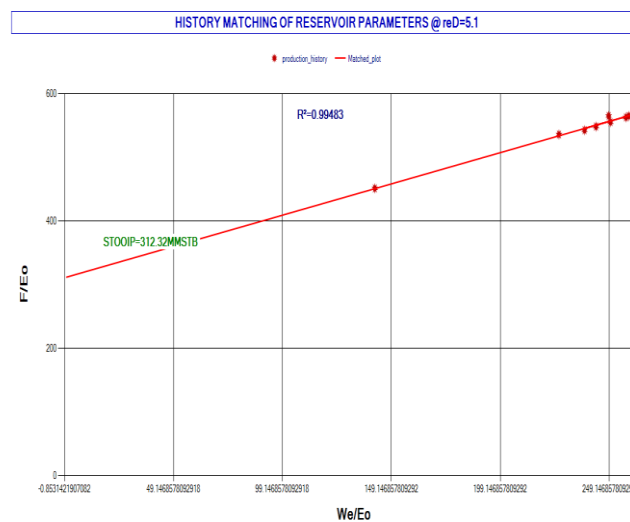


Fig. 9. History matching and reservoir volume estimation using QUANTIFY.

3.1. Model validation

Despite the reality of the results presented, it is paramount that the computer model used in this study is validated to guarantee a reduced computational error. Two methods were used to validate the results of this research. One method is the R-squared determination. This method refers to the ratio of explained variation to the total variation. That being said, the R-squared value was estimated to be 0.99483 – which is very acceptable. The second validation method employed in this study was the commercial software (MBAL) used to perform similar runs as the QUANTIFY software. Material balance plot to determine OOIP for the same reservoir was run on MBAL, and results from the commercial tool can be seen in Figure 10 and Figure 11.

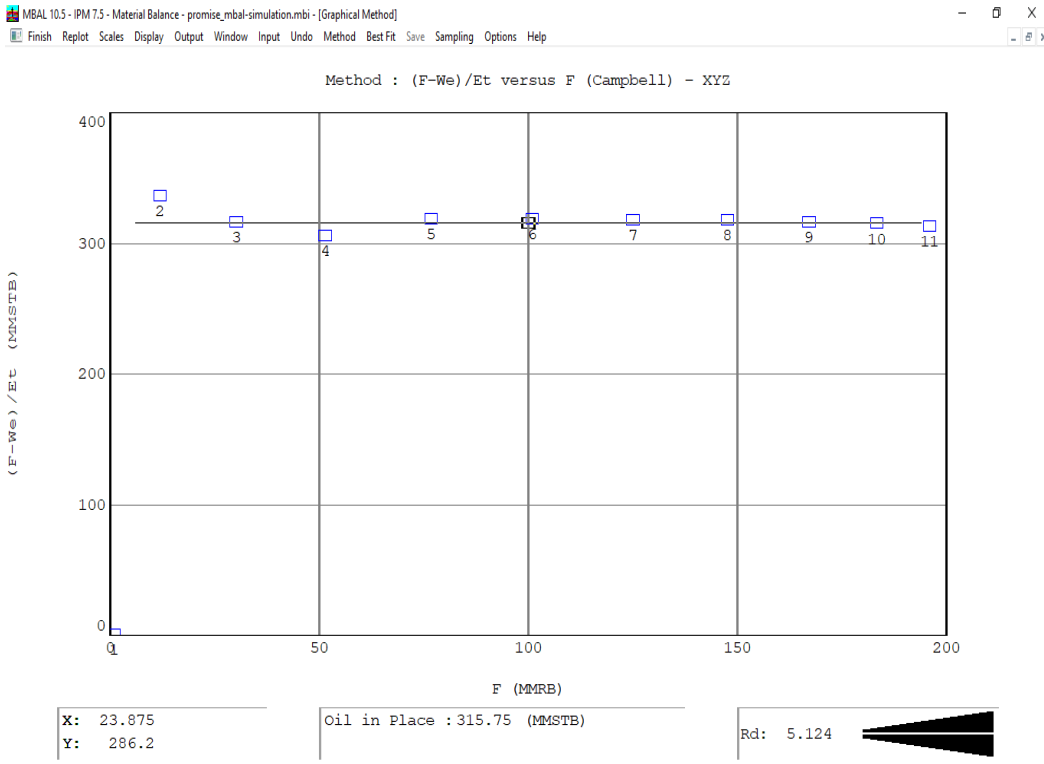


Fig. 10. OOIP Graphical plot using MBAL.

A STOOIP of 315.75 MMSTB was estimated using MBAL. Figure 11 shows the match points for the reservoir with and without an aquifer. Nonetheless, it was observed that the data points fitted perfectly at a permeability of 327md and r_{eD} of 5.134 with a STOOIP of 315.75 MMSTB.

The energy drive plot obtained from the MBAL tool is also presented in Figure 12 and was observed to have a similar trend to that obtained from QUANTIFY software. However, the driving plot provided by QUANTIFY software appears with better visual quality than that of MBAL because it was presented in 3D; meanwhile, the MBAL energy drive plot was in 2D.

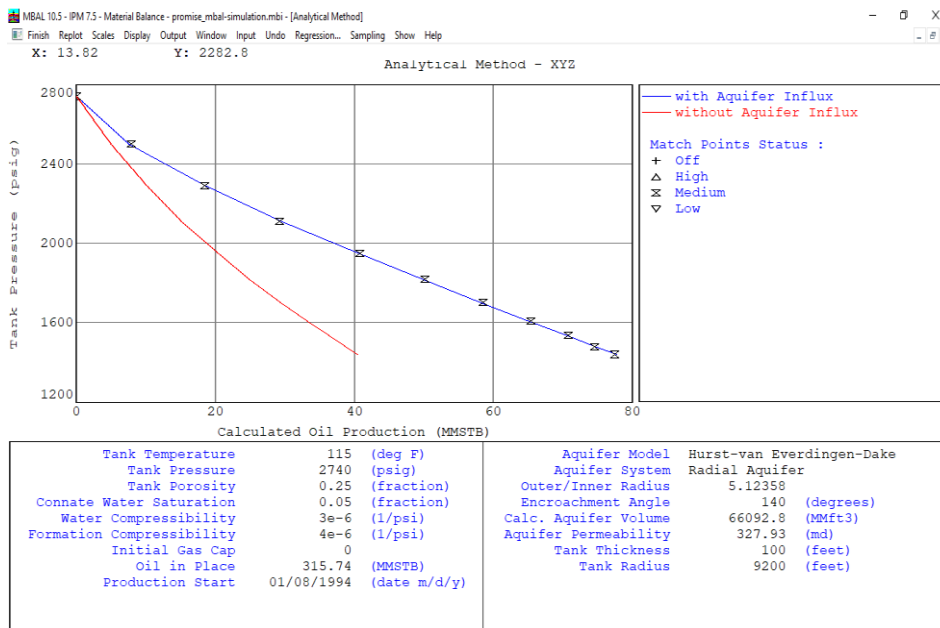


Fig. 11. History matching with or without water influx using MBAL.

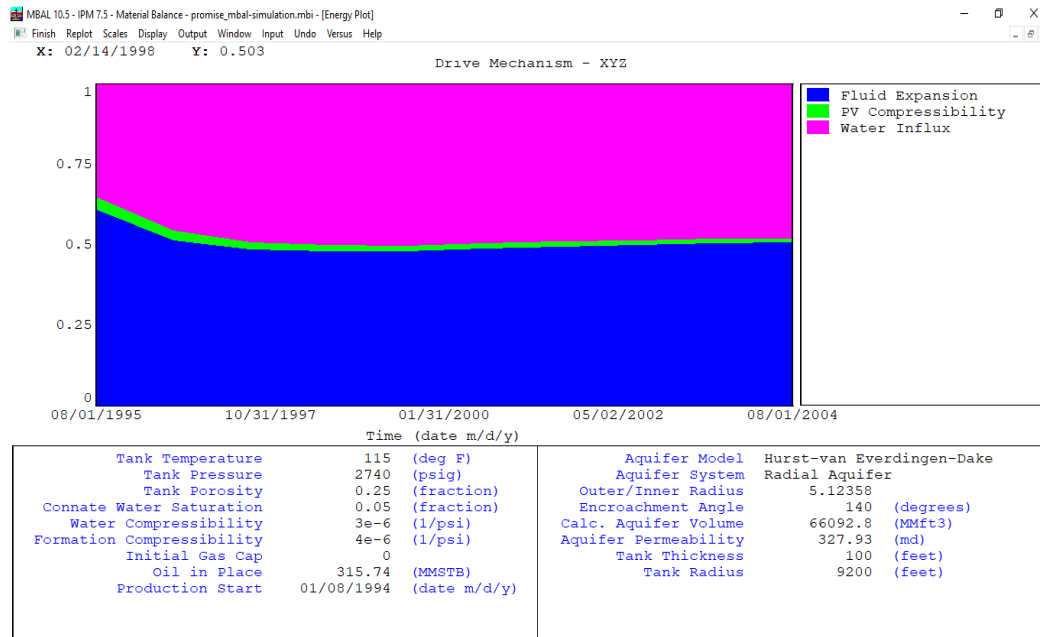


Fig. 12. Energy drive plot using MBAL.

More so, a comparative analysis was performed on results derived from MBAL, QUANTIFY and published results by Dake [11-12]. The comparison was performed using Dake's result as a base case while calculating the percentage error. The result is shown in Table 6. Results from the comparison showed that QUANTIFY tool gave a more accurate result than MBAL with lesser percentage errors. Although results from both tools are very similar, those obtained from QUANTIFY tool proved to be more precise.

Table 6. Comparative analysis between MBAL, QUANTIFY and LP DAKE [11-12].

PARAMETER	QUANTIFY	MBAL	DAKE	%ERROR- QUANTIFY	%ERROR- MBAL
	<i>Hurst-Van Everdingen</i>	<i>Hurst-Van Everdingen-Dake</i>	<i>Hurst-Van Everdingen</i>		
Reservoir Thickness (ft)	100	100	100	0	0
Reservoir Radius (ft)	9200	9200	9200	0	0
Outer/Inner Radius (r _{ED})	5.1	5.124	5	2	2.48
Encroachment Angle	140	140	140	0	0
Aquifer Permeability (md)	200	327.93	200	0	63.965
OIIP (MMSTB)	312.32	315.75	312	0.102564103	1.201923077

4. CONCLUSIONS

This study successfully demonstrated that water influx parameters (W_D and W_e) could be easily determined using the complex Van-Everdingen and Hurst Model. After water influx calculation, reserve estimation can be performed using the straight-line material balance method. This study used a least square regression model to express MBE as a straight line. Moreover, published data were used to test the validity of the developed model QUANTIFY. To determine the nature of the reservoir energy drive, the "developed tool" performed an energy drive plot in 3D for a qualitative visualisation. It was then confirmed that the reservoir was primarily driven by water influx and fluid expansion. Several runs were made using QUANTIFY tool to obtain the STOOIP of the reservoir with the most minimal error. A simulation run at an outer/inner radius of 5.1 gave a STOOIP of 312.32 MMSTB at an R^2 value

of 0.99483 – which was the closest value to 1 of all the simulations run on the QUANTIFY tool. Therefore, the oil originally in place (OOIP) was determined to be 312.32 MMSTB.

To validate the results obtained from the developed tool (QUANTIFY), a commercial tool MBAL was used to calculate STOOIP using the same data. OOIP was estimated to be 315.93 MMSTB, similar to the result obtained from QUANTIFY software. A comparative analysis was performed on MBAL, QUANTIFY and DAKE results, using Dake's result as a base/correct case. The percentage error in OOIP for QUANTIFY software was estimated as 0.1%; meanwhile, that of MBAL was estimated as 1.2%. Invariably, the percentage errors were lesser for QUANTIFY software than for MBAL.

This study has shown that “Water Influx” parameters can be determined through appropriate correlations computationally without using charts or tables. The study has further confirmed the efficacy that reserve estimation for an aquifer-supported reservoir can be performed using Van Everdingen and Hurst aquifer model. This study used a 3D reservoir energy drive plot to visualize reservoir drive mechanisms better.

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